Kriptografik İspat Sistemlerinin Gelişim Serüveni

EPFL

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R1CS

PLONK

Aurora

STARK

HyperPlonk

TurboPLONK

cq

Pinocchio

Nova

Sangria

Breakdown

FRI

Caulk

Groth16

AIR

HyperNova

Baloo

CCS

Halo2

SuperNova

Caulk+

Bulletproofs

KZG

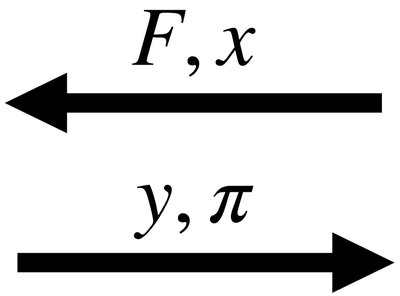
ProtoStar

General Purpose Verifiable Computation

Task: Compute $F(x)$

$F(x) \rightarrow y$

$Prove(F, x, y) \rightarrow \pi$



$Verify(F, x, y, \pi) \rightarrow 0/1$

General Purpose Verifiable Computation

Task: Compute $F(x, w)$

F, x

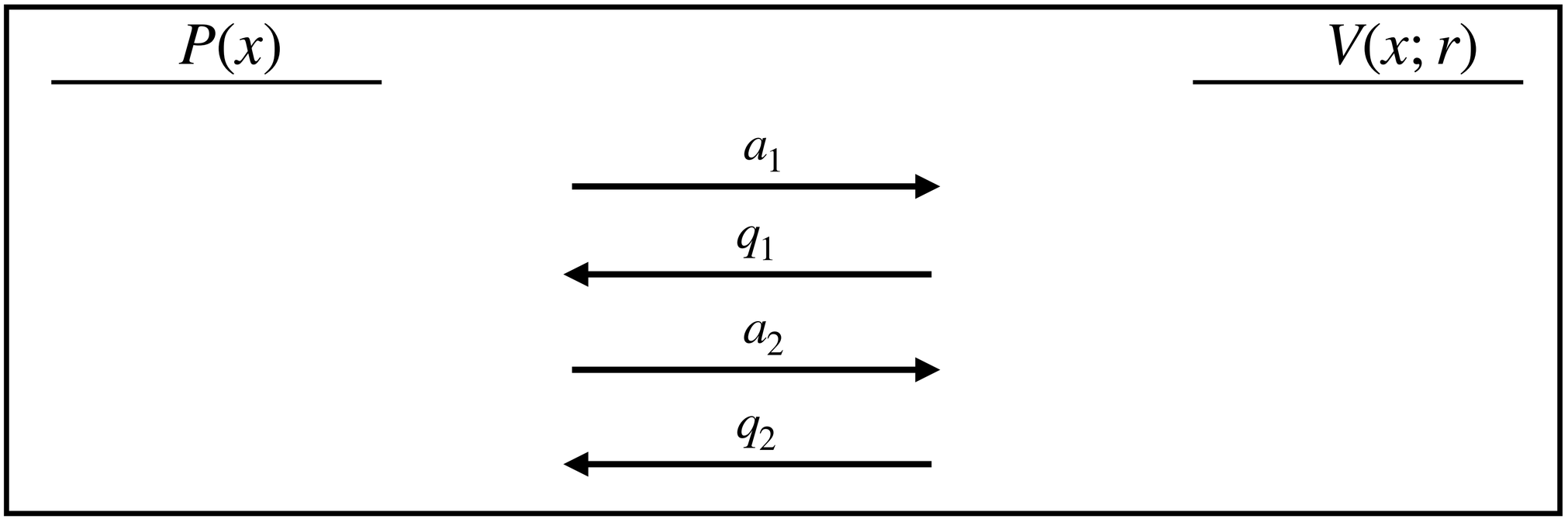
y, π

$F(x, w) \rightarrow y$

$Prove(F, x, y) \rightarrow \pi$

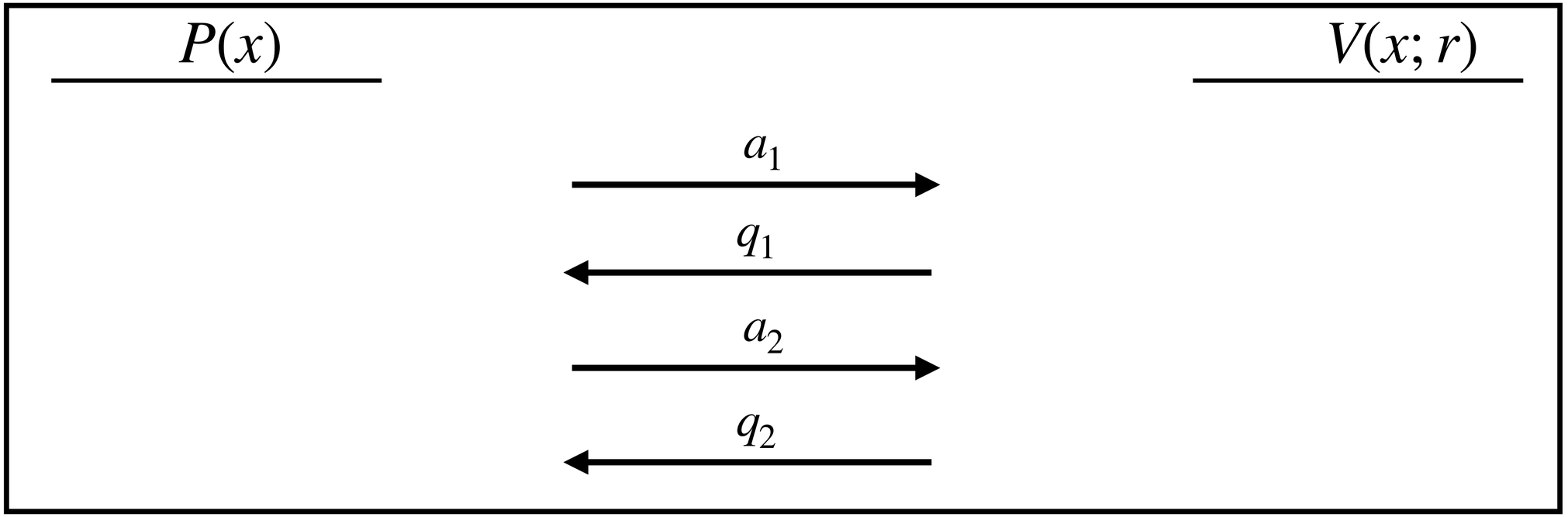
$Verify(F, x, y, \pi) \rightarrow 0/1$

Interactive Proof



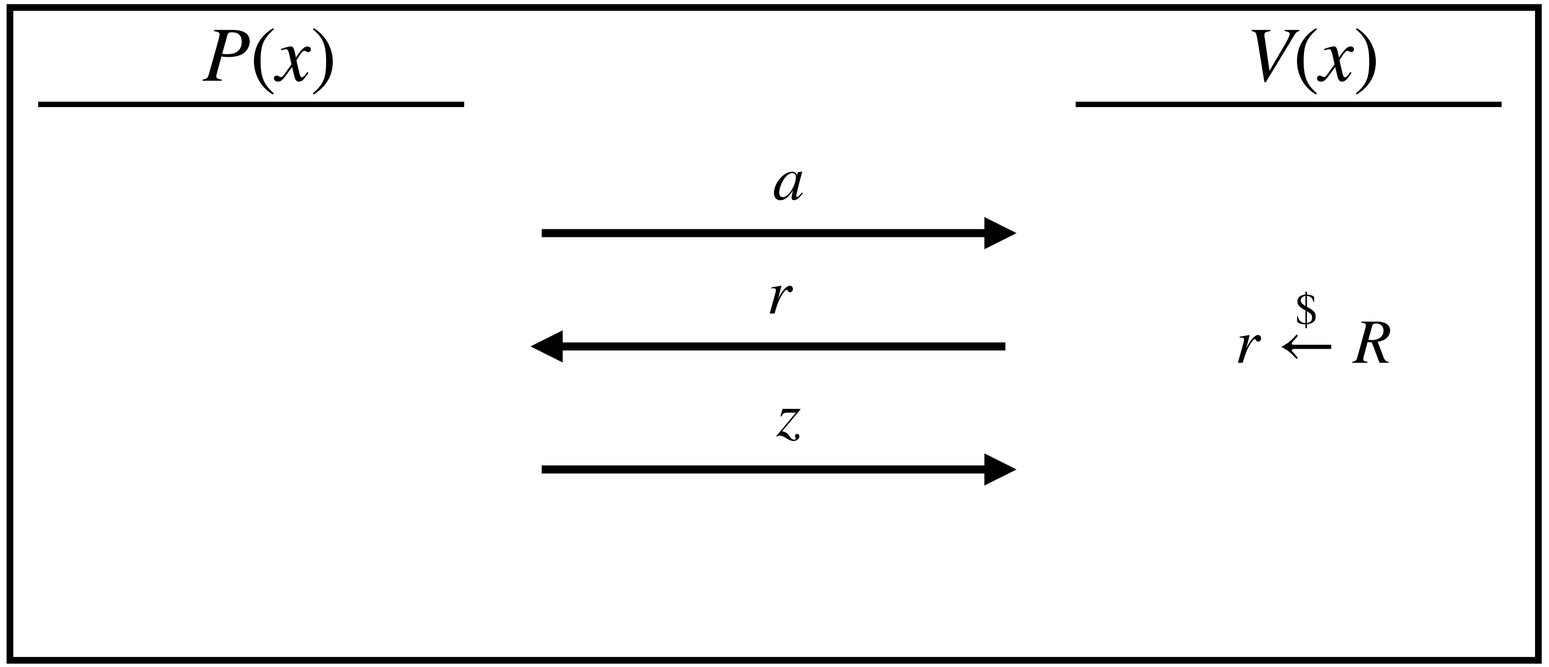
- **Completeness:** $\forall x \in L \qquad \Pr_r[\langle P(x), V(x; r)\rangle = 1] = 1$
- **Soundness:** $\forall x \notin L \quad \forall P^* \qquad \Pr_r[\langle P^*(x), V(x; r)\rangle = 1] \leq 1/2$

Interactive Argument



- **Completeness:** $\forall x \in L \qquad \Pr_r[\langle P(x), V(x; r)\rangle = 1] = 1$
- **Soundness:** $\forall x \notin L \;\; \forall \text{ PPT } P^* \quad \Pr_r[\langle P^*(x), V(x; r)\rangle = 1] \leq 1/2$

Sigma Protocol



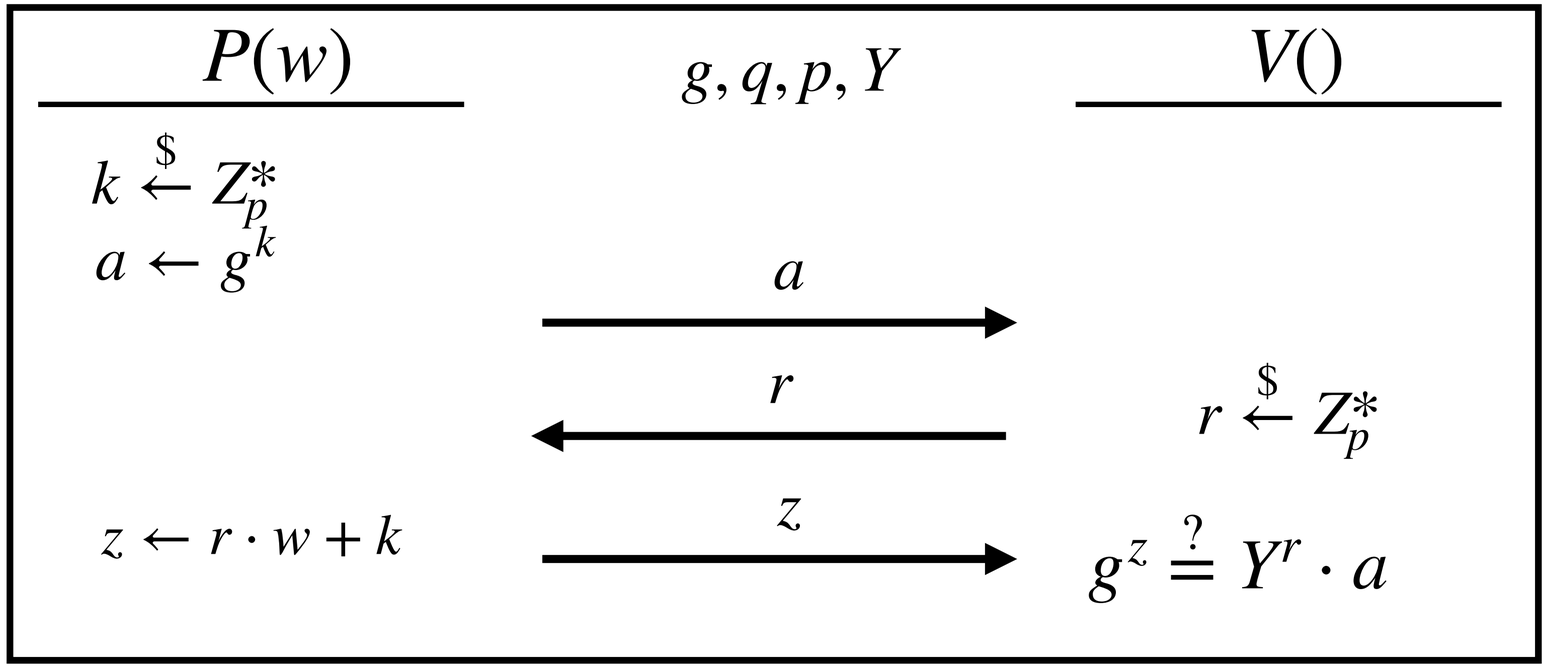
Sigma Protocol
DLOG

Let g be the generator of a subgroup of large prime order q modulo p .

Prover: I know w such that $Y = g^w \mod p$

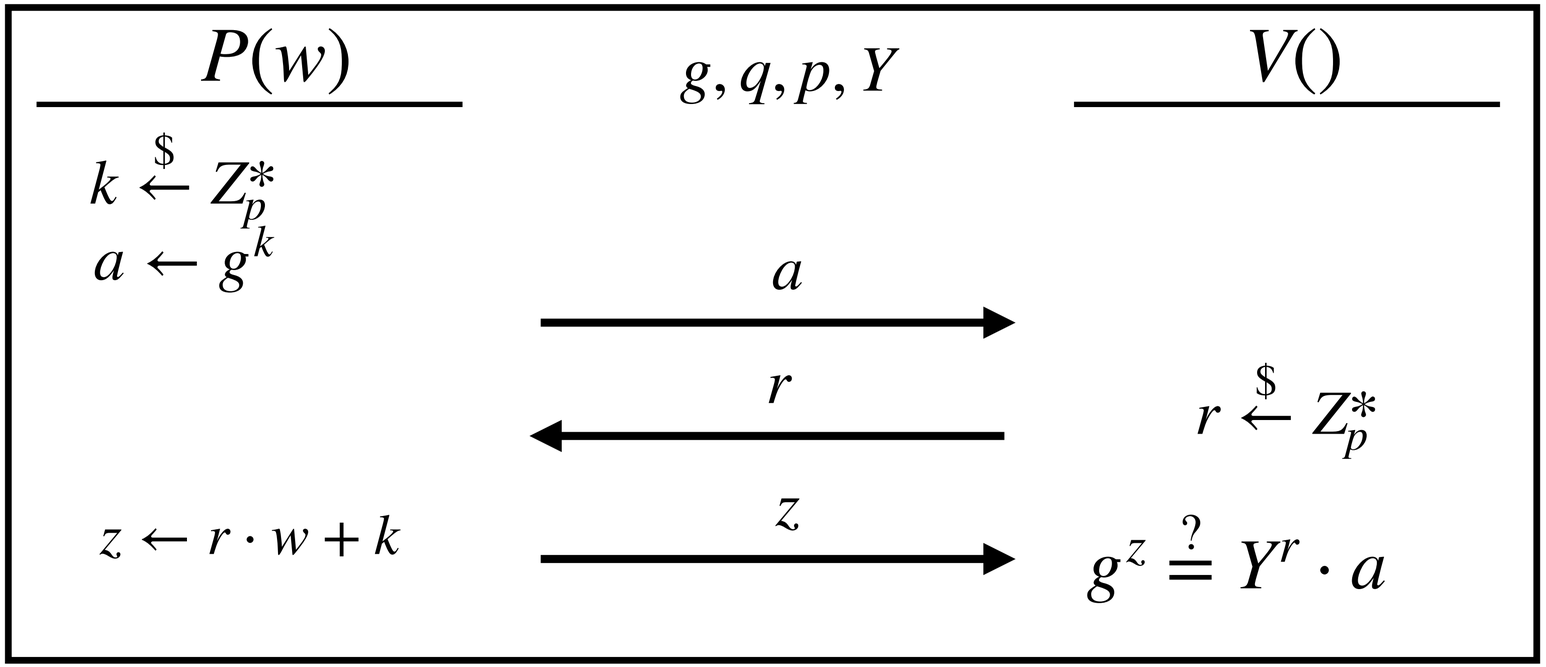
Sigma Protocol

Schnorr DLOG



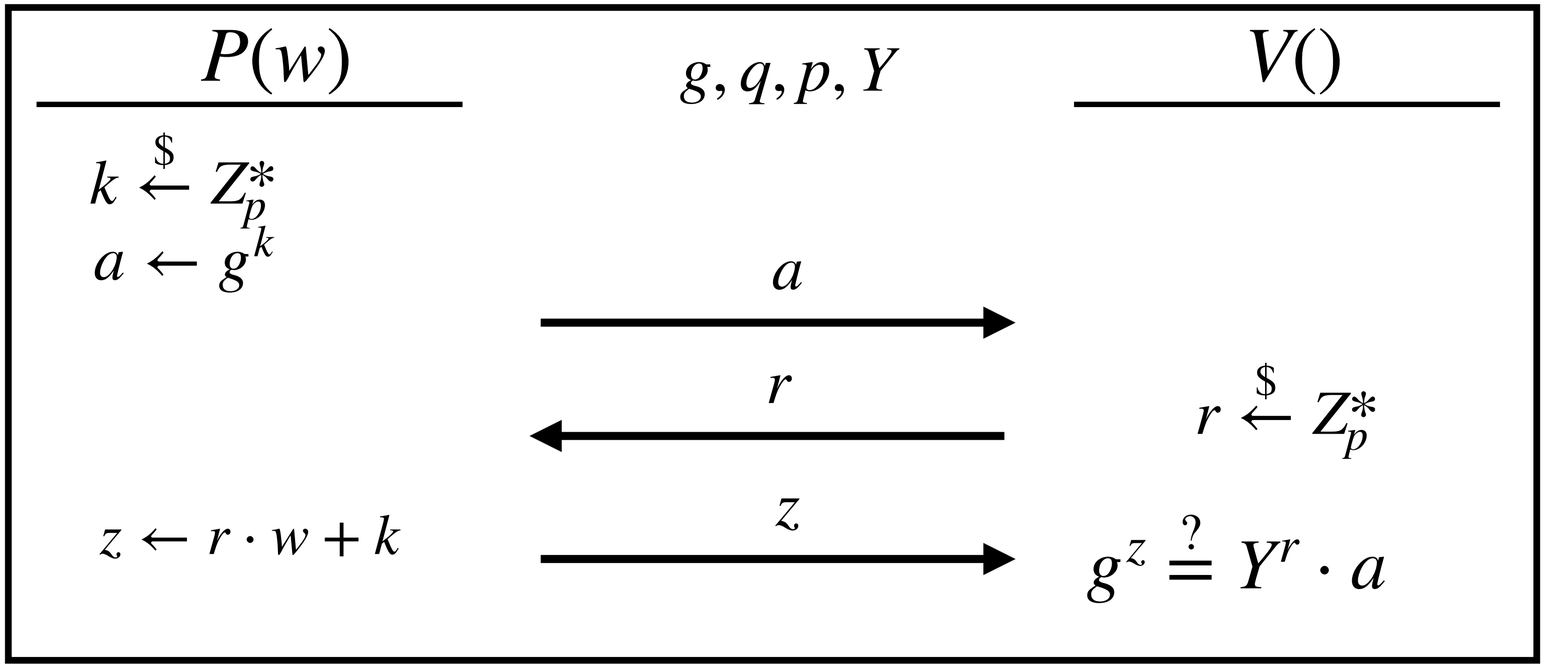
Sigma Protocol

Schnorr DLOG



- **Completeness**
- **Knowledge Soundness**
- **HV Zero-Knowledge**

Sigma Protocol
DLOG



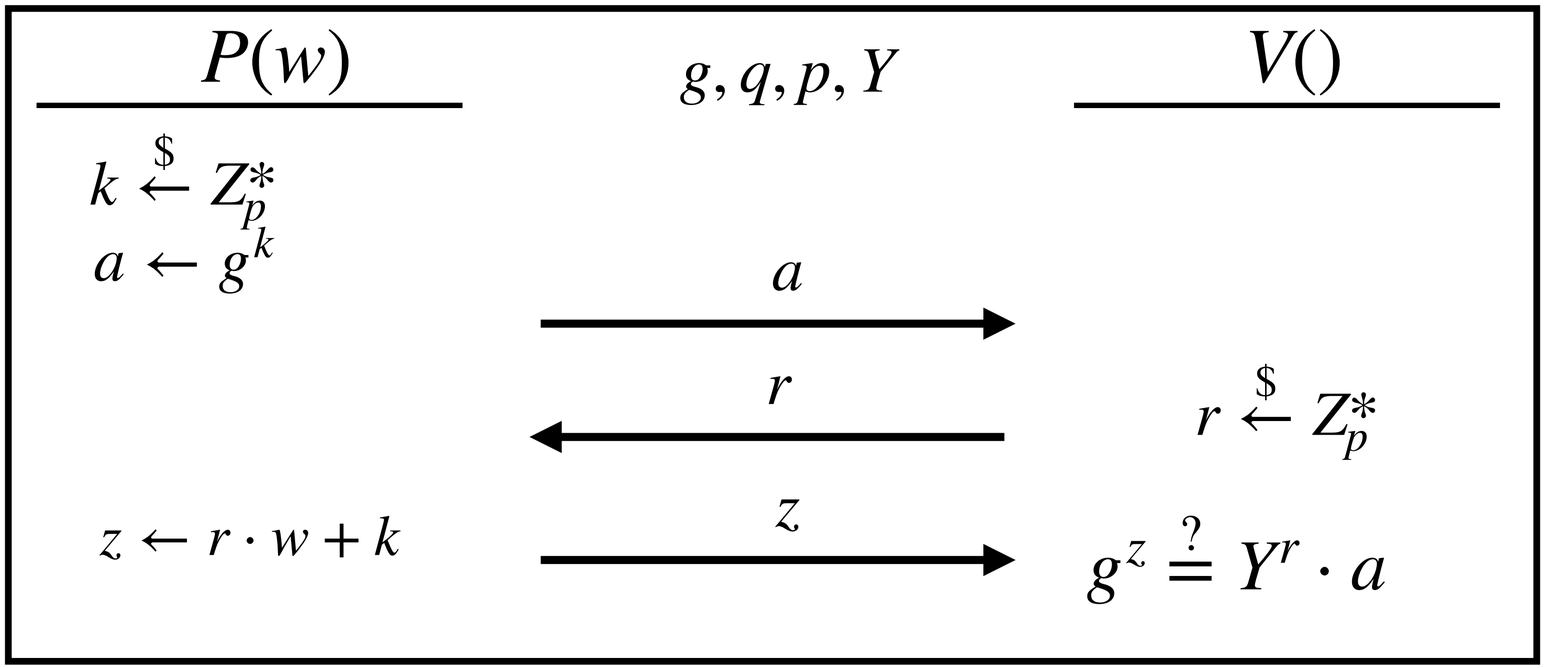
- **Completeness:**

$$g^z \stackrel{?}{=} Y^r \cdot a$$

$$g^{(r \cdot w + k)} \stackrel{?}{=} (g^w)^r \cdot g^k$$

$$g^{r \cdot w + k} = g^{w \cdot r + k}$$

Sigma Protocol
DLOG



- **Knowledge Soundness**

Given (a, r, z) and (a, r', z')

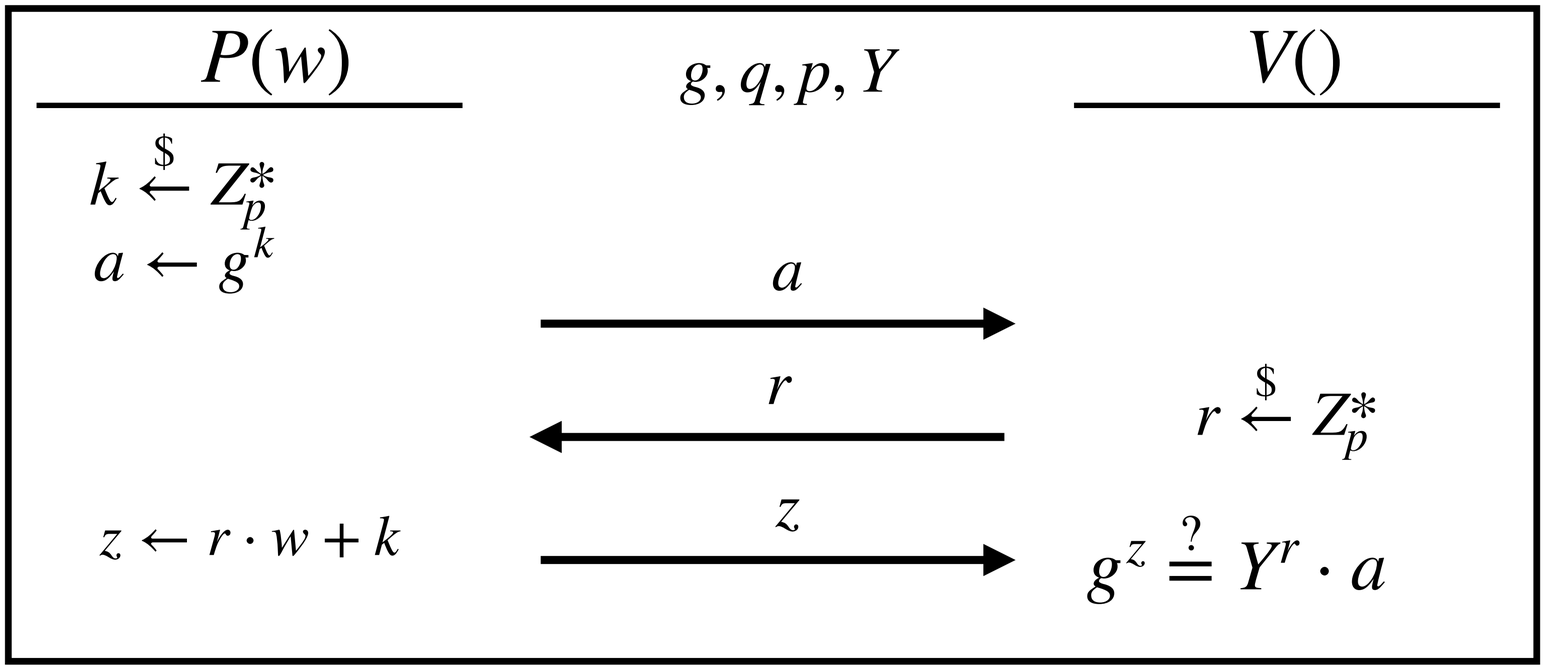
as valid transcripts.

Extract w in poly time.

$$g^z = Y^r \cdot a$$

$$g^{z'} = Y^{r'} \cdot a$$

Sigma Protocol
DLOG



- **Knowledge Soundness**

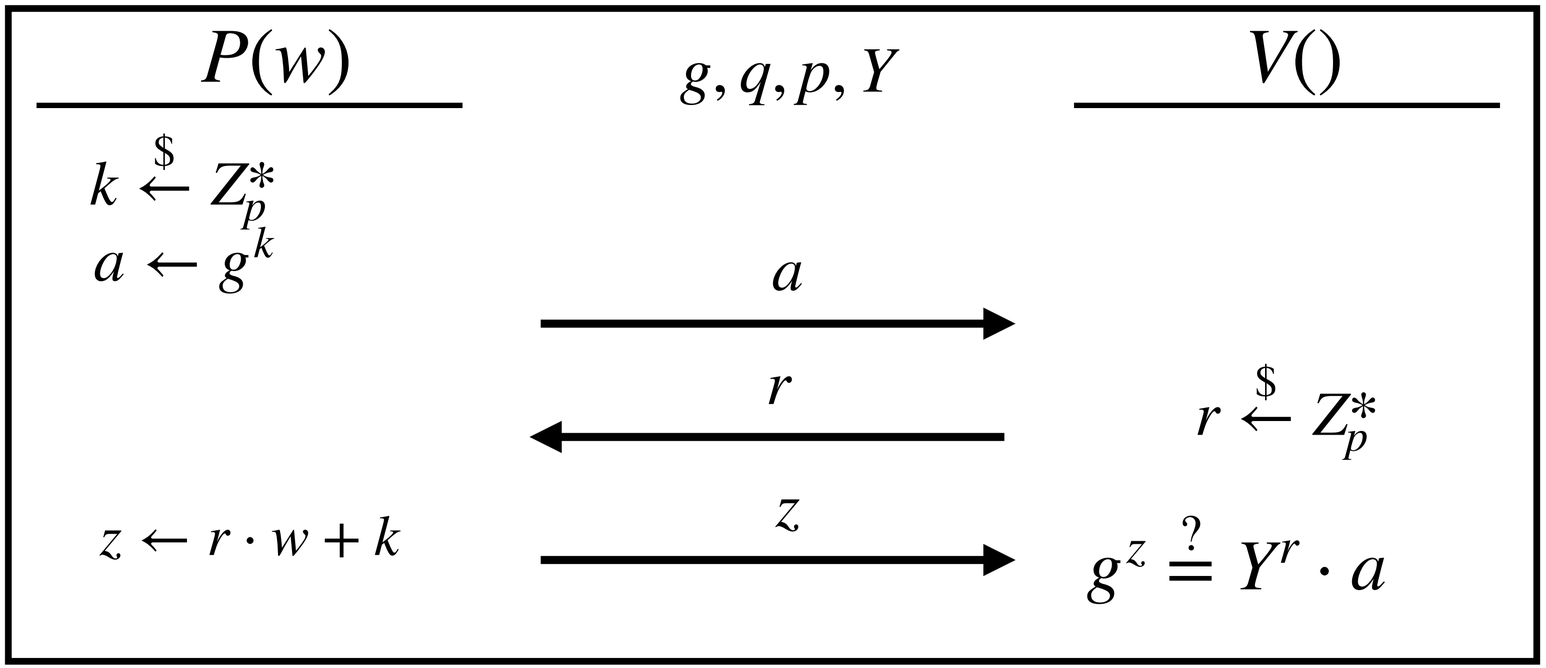
Given (a, r, z) and (a, r', z')

as valid transcripts.

Extract w in poly time.

$$g^z = Y^r \cdot a$$
$$g^{z'} = Y^{r'} \cdot a$$
$$\Longrightarrow \quad g^{(z-z')} = Y^{(r-r')}$$

Sigma Protocol
DLOG



- **Knowledge Soundness**

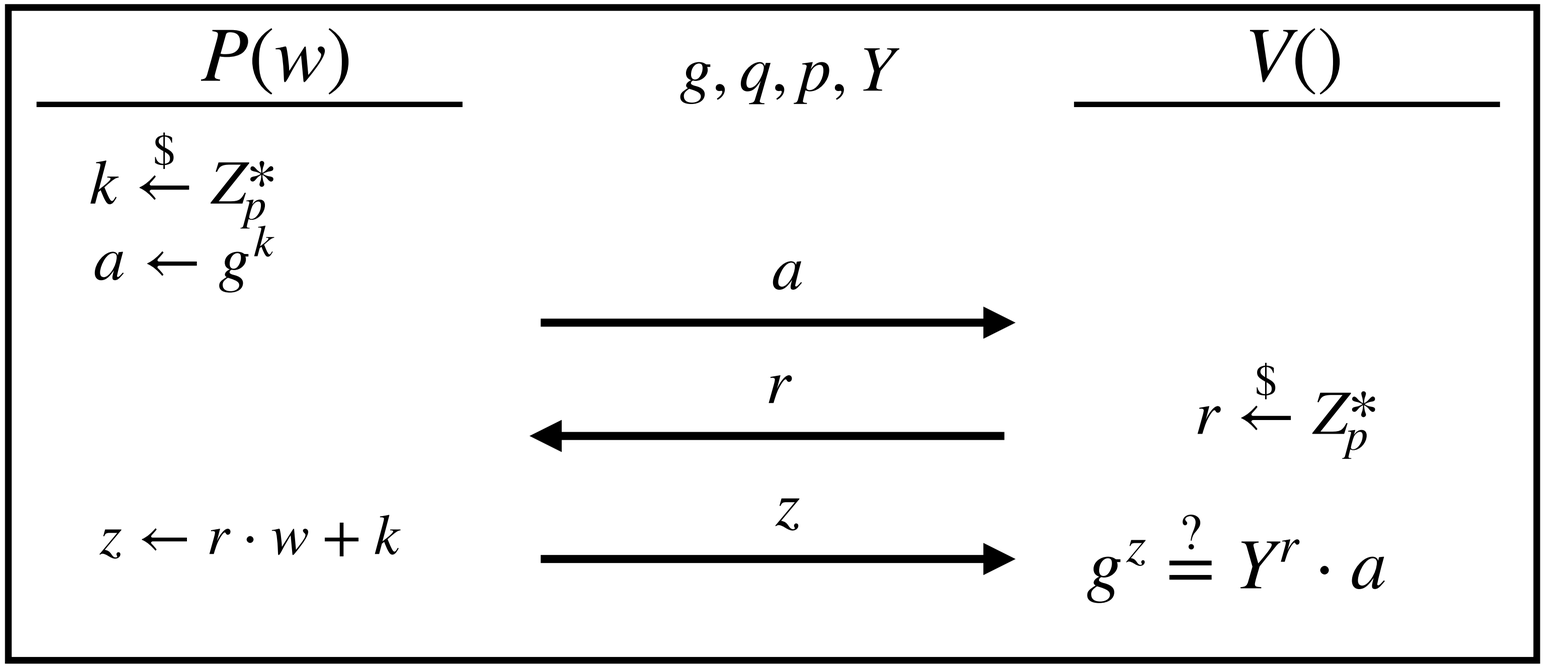
Given (a, r, z) and (a, r', z')

as valid transcripts.

Extract w in poly time.

$$g^z = Y^r \cdot a$$
$$g^{z'} = Y^{r'} \cdot a$$
$$\implies g^{(z-z')} = Y^{(r-r')} \implies g^{(z-z')/(r-r'))} = Y$$
$$\implies w = (z - z')/(r - r')$$

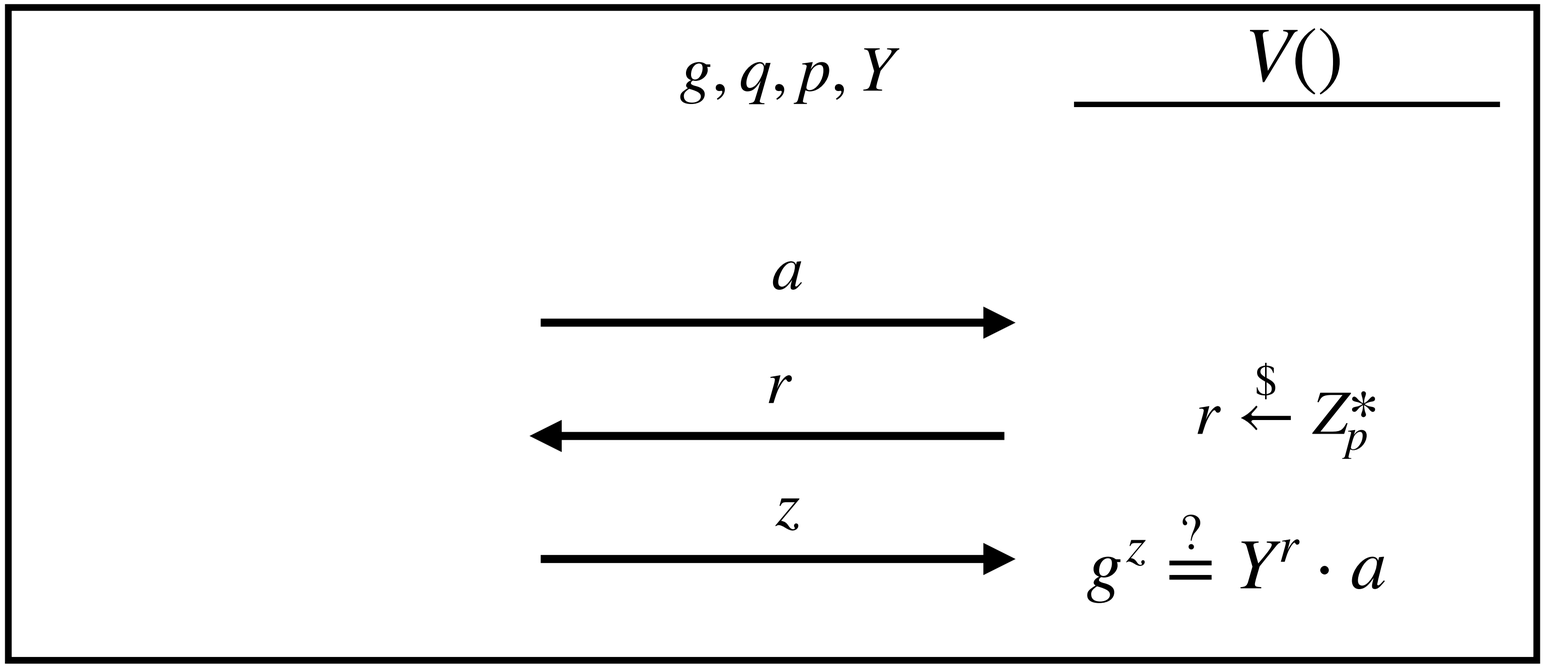
Sigma Protocol
DLOG



- **HV Zero-Knowledge**

A valid transcript can be efficiently simulated.

Sigma Protocol
DLOG



- **HV Zero-Knowledge**

A valid transcript can be efficiently simulated.

Sigma Protocol
DLOG

$S()$	g, q, p, Y	$V()$
$z, r \xleftarrow{\$} Z_p^*$		
$a \leftarrow g^z \cdot Y^{-r}$	(a, r, z)	$g^z \stackrel{?}{=} Y^r \cdot a$

- **HV Zero-Knowledge**

A valid transcript can be efficiently simulated.

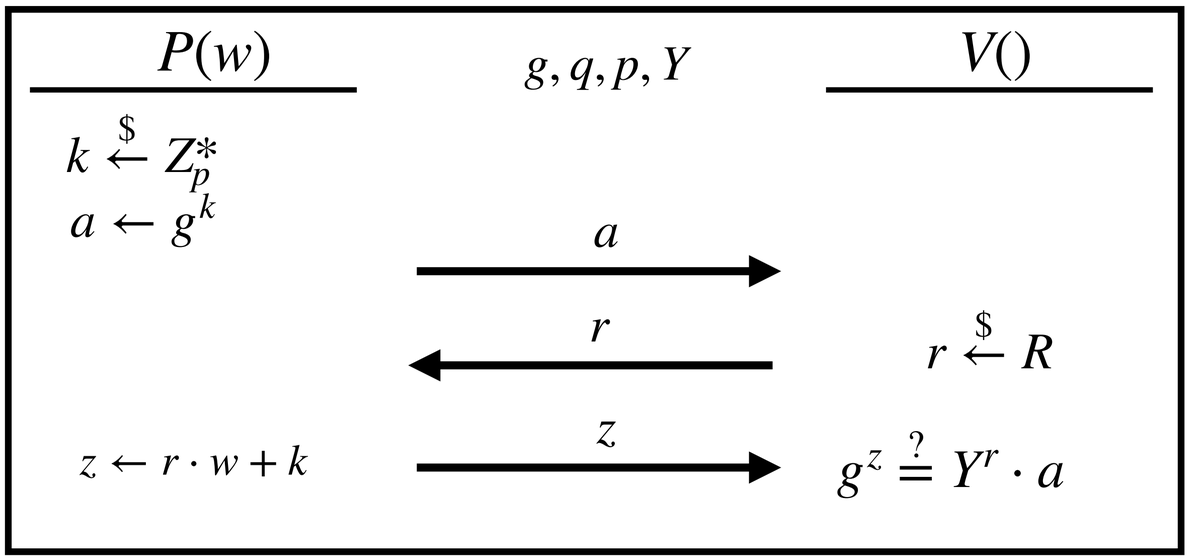
Sigma Protocol
ROM

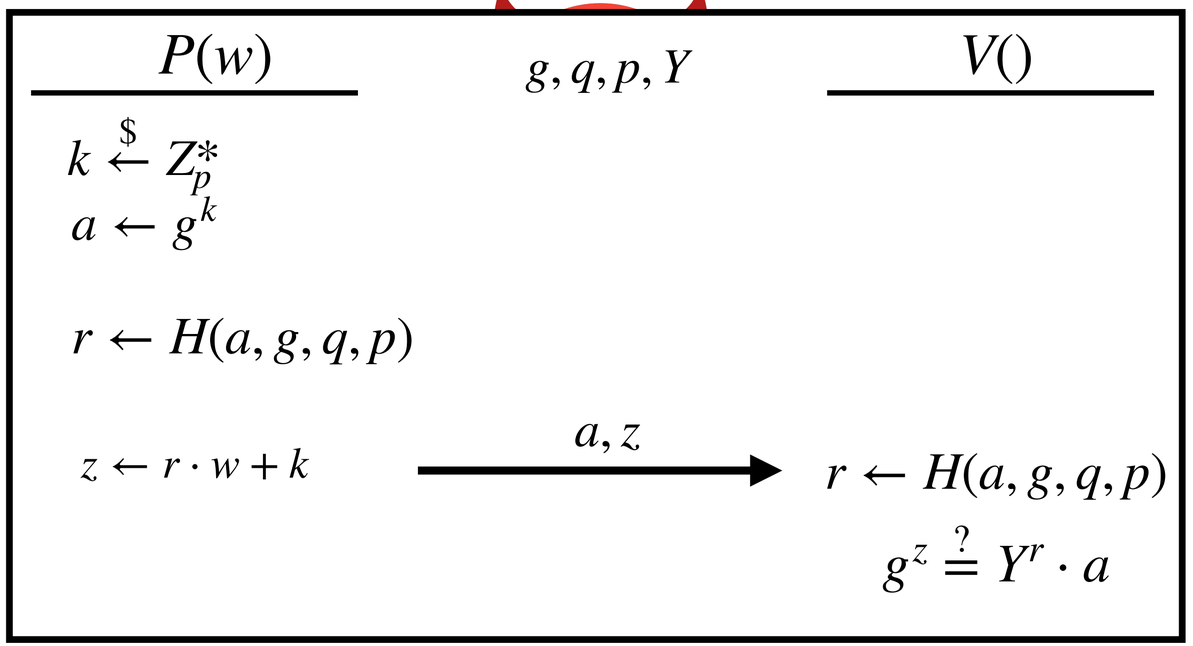
Blackbox oracle $H(\ .\ )$ that returns consistent but uniformly random values.

Realized using a hash function e.g. SHA256

Sigma Protocol

Non-interactivity via Fiat Shamir Heuristic





General Purpose Verifiable Computation

Task: Compute $F(x)$

F, x

y, π

$F(x) \rightarrow y$

$Prove(F, x, y) \rightarrow \pi$

$Verify(F, x, y, \pi) \rightarrow 0/1$

General Purpose Verifiable Computation

Succinct **N**on-interactive **ARG**ument

Soundness: There exists w such that $F(x, w) = y$

General Purpose Verifiable Computation

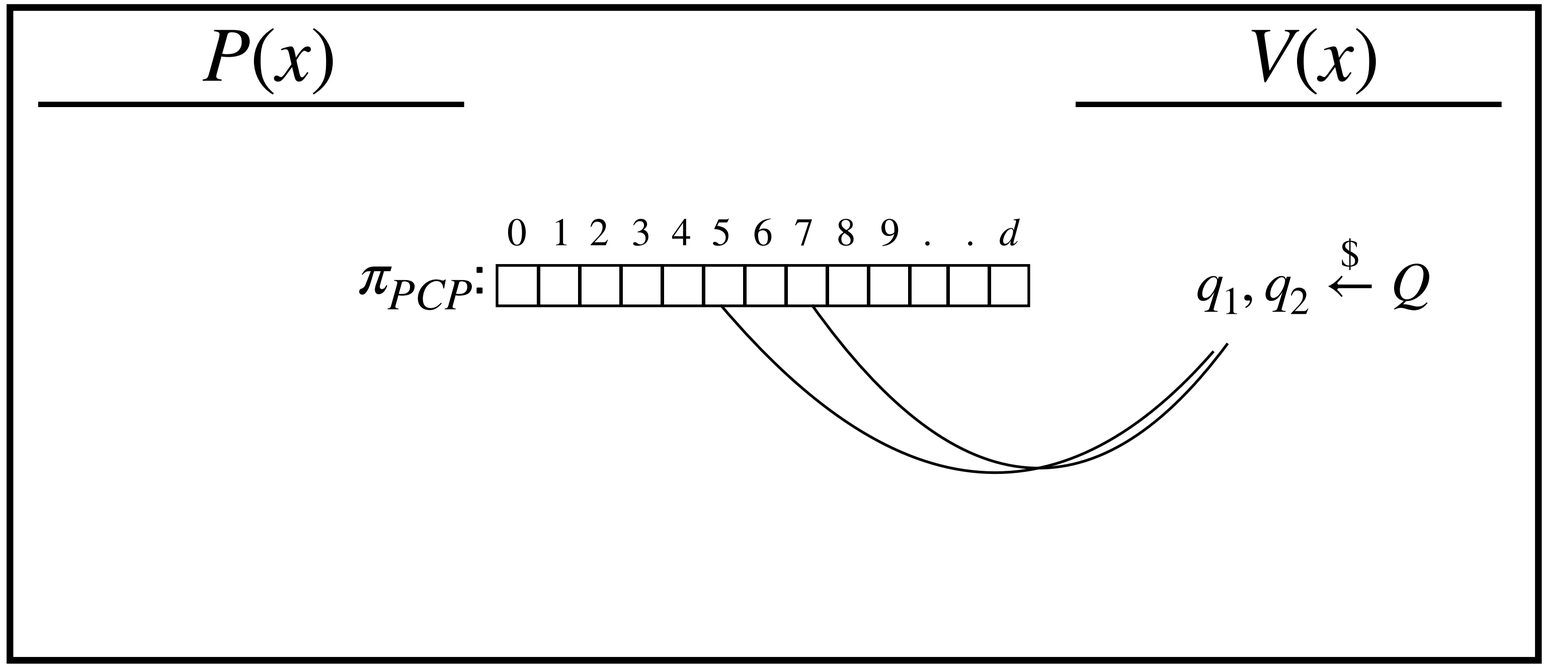
Succinct **N**on-interactive **AR**gument of **K**nowledge

Knowledge Soundness:

There exists w **known by the prover** such that $F(x, w) = y$

PCP

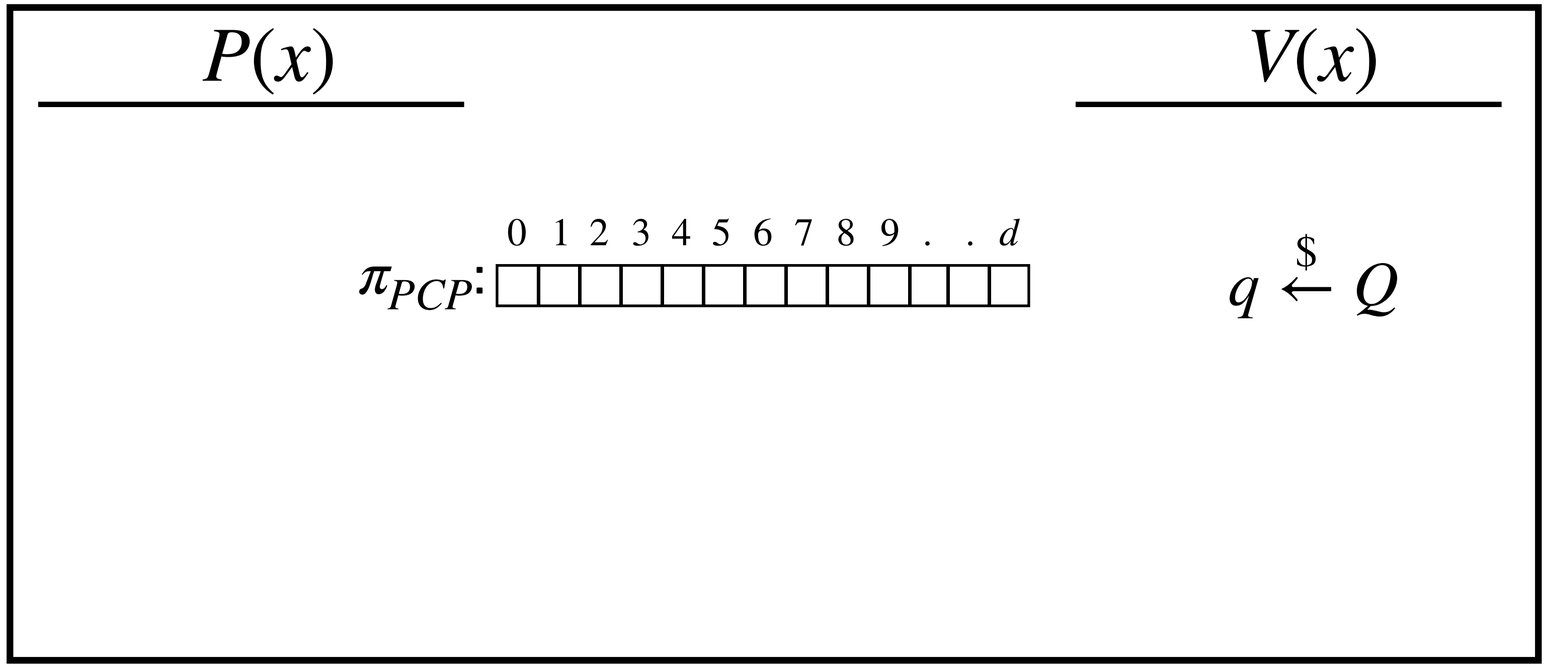
[BFLS'91]



"In this setup, a single reliable PC can monitor the operation of a herd of supercomputers working with possibly extremely powerful but unreliable software and untested hardware."

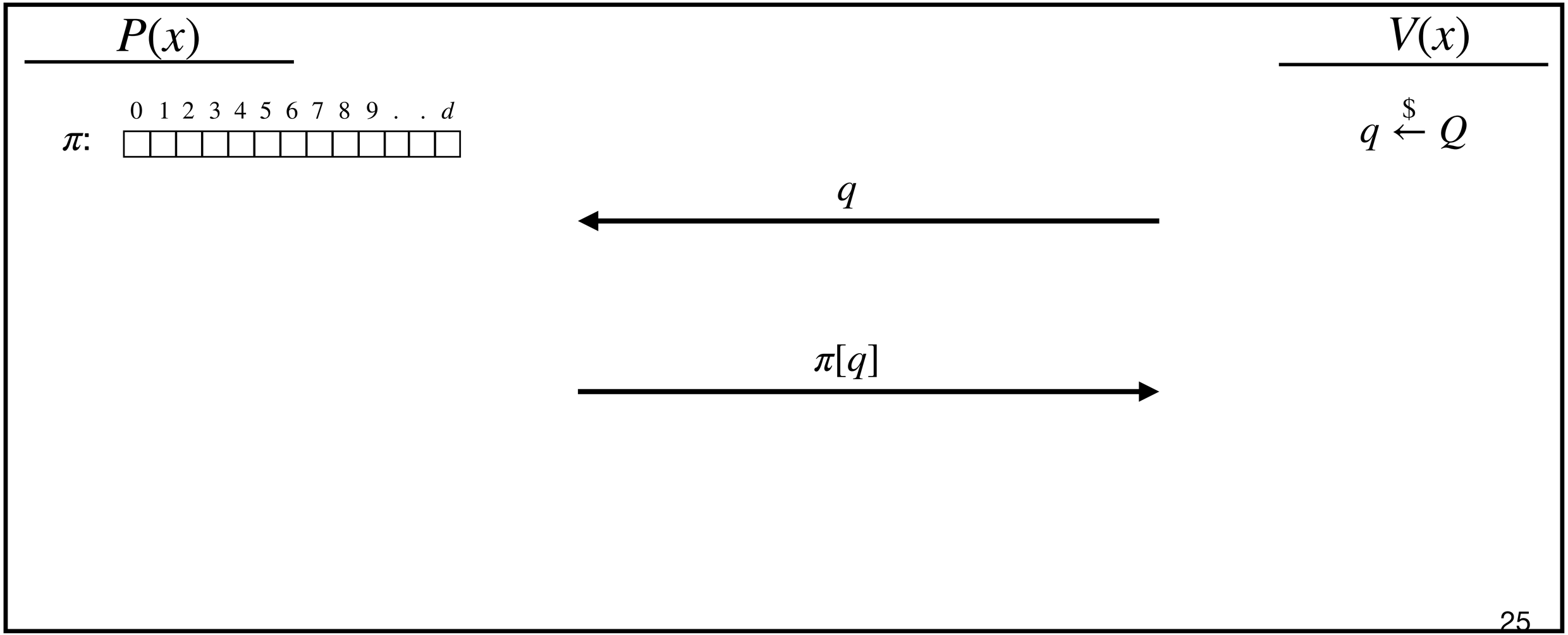
PCP

[Killian'92]



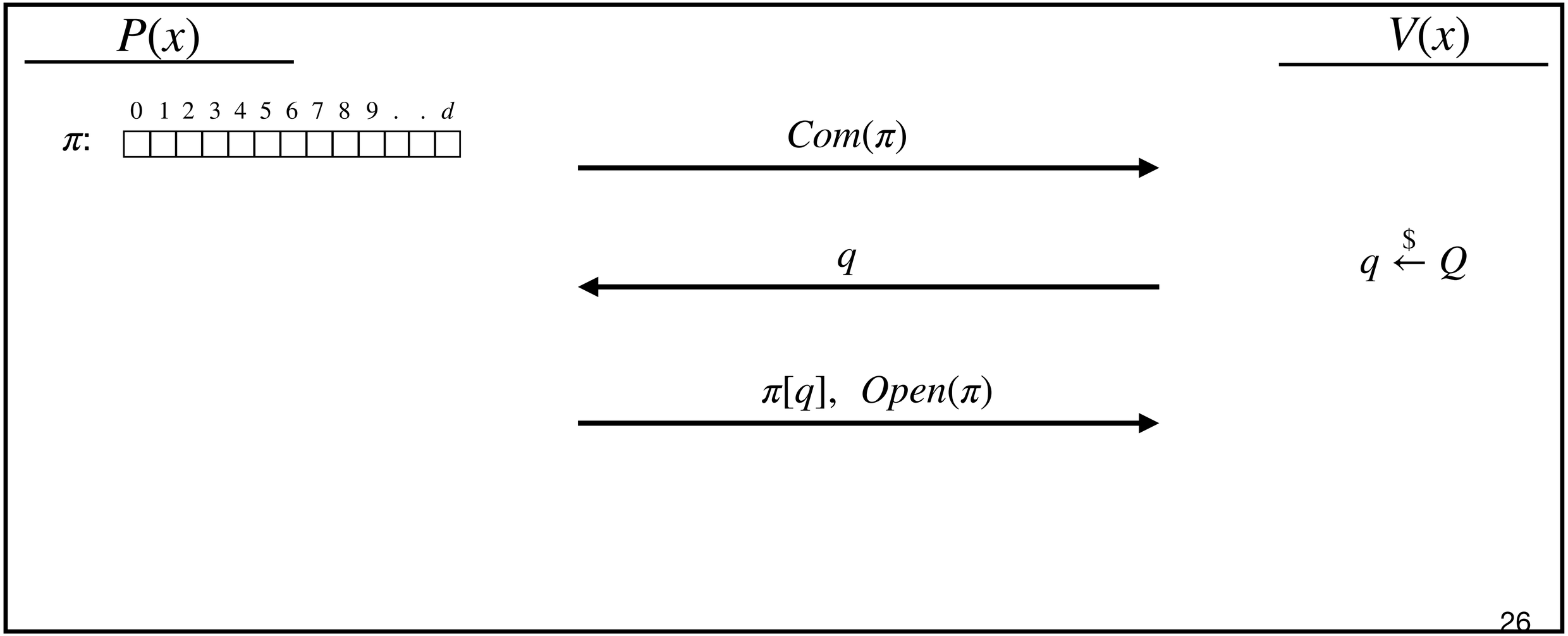
PCP

[Killian'92]



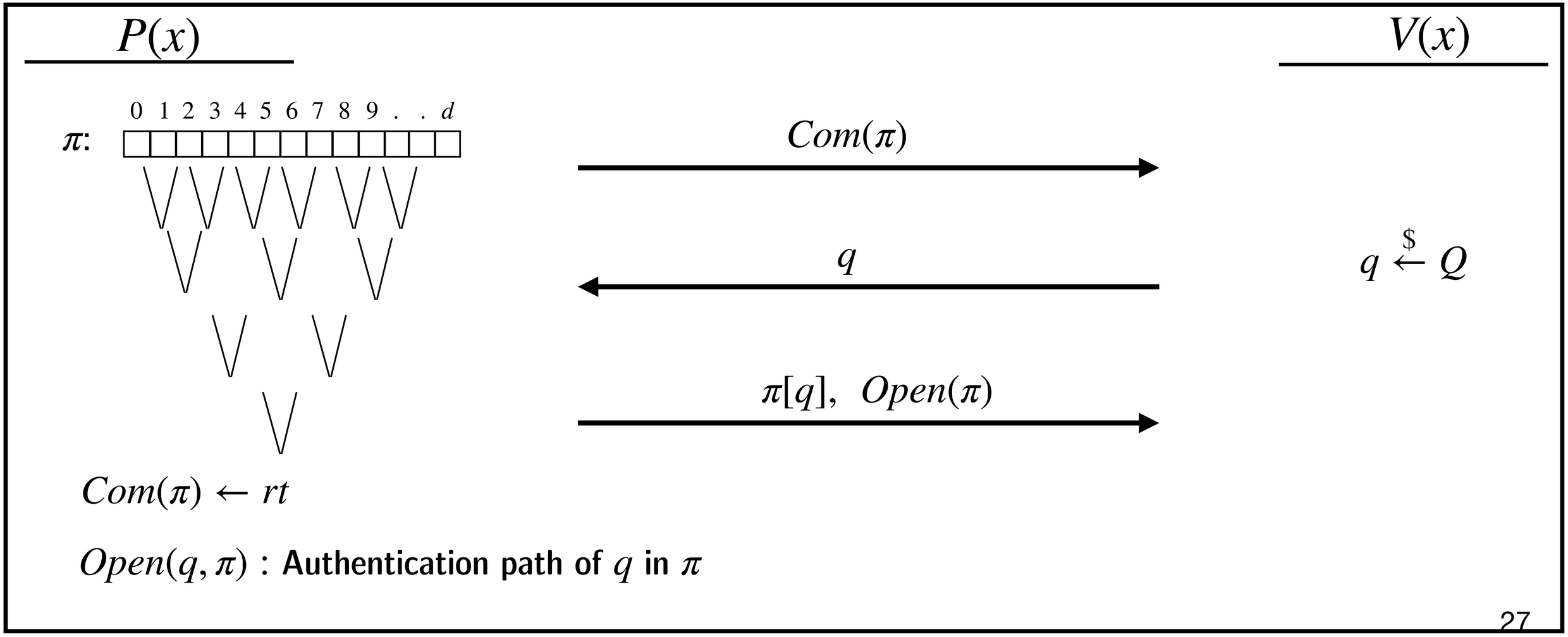
PCP

[Killian'92]



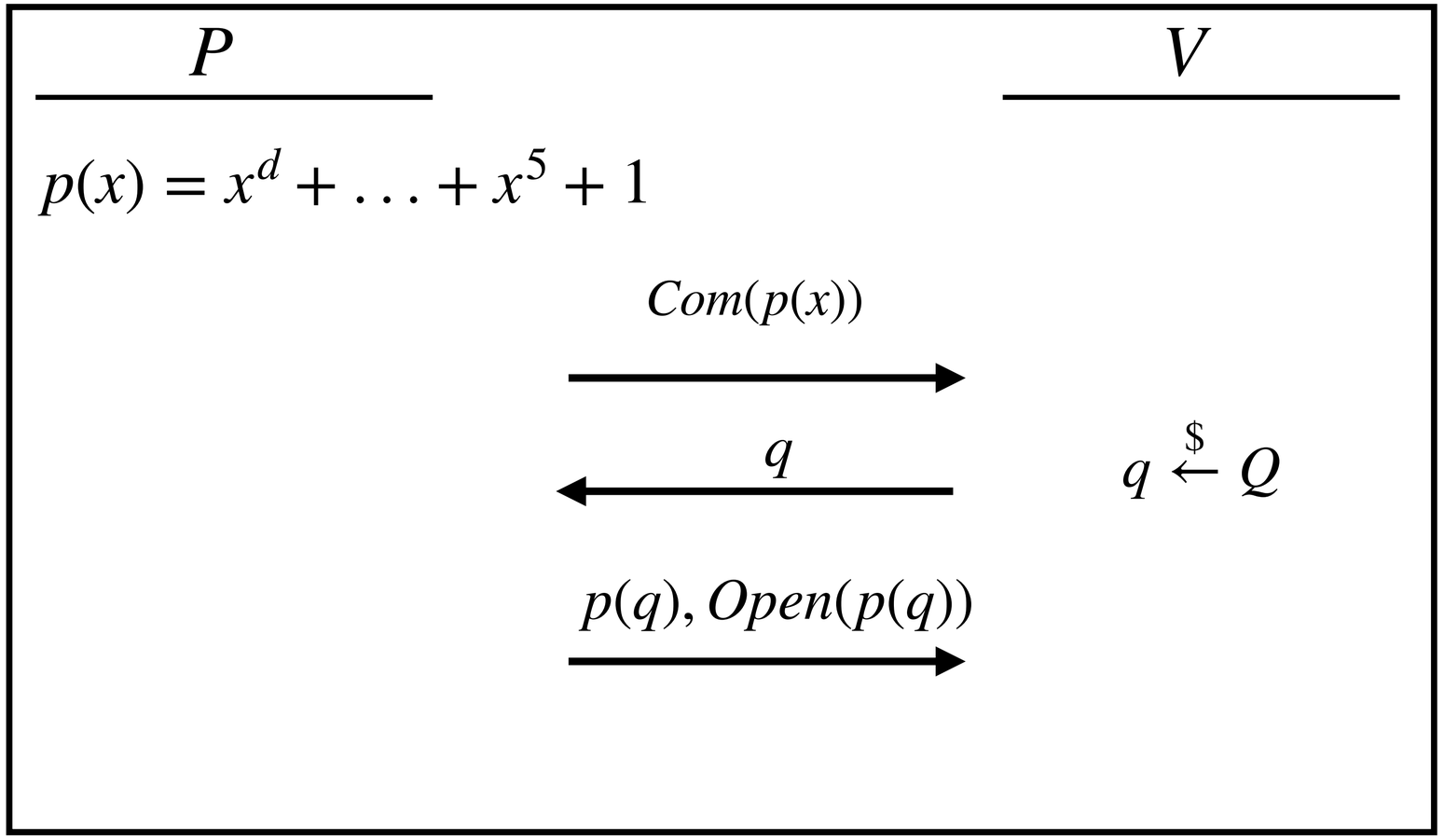
PCP

[Killian'92] Use a Vector Commitment to π



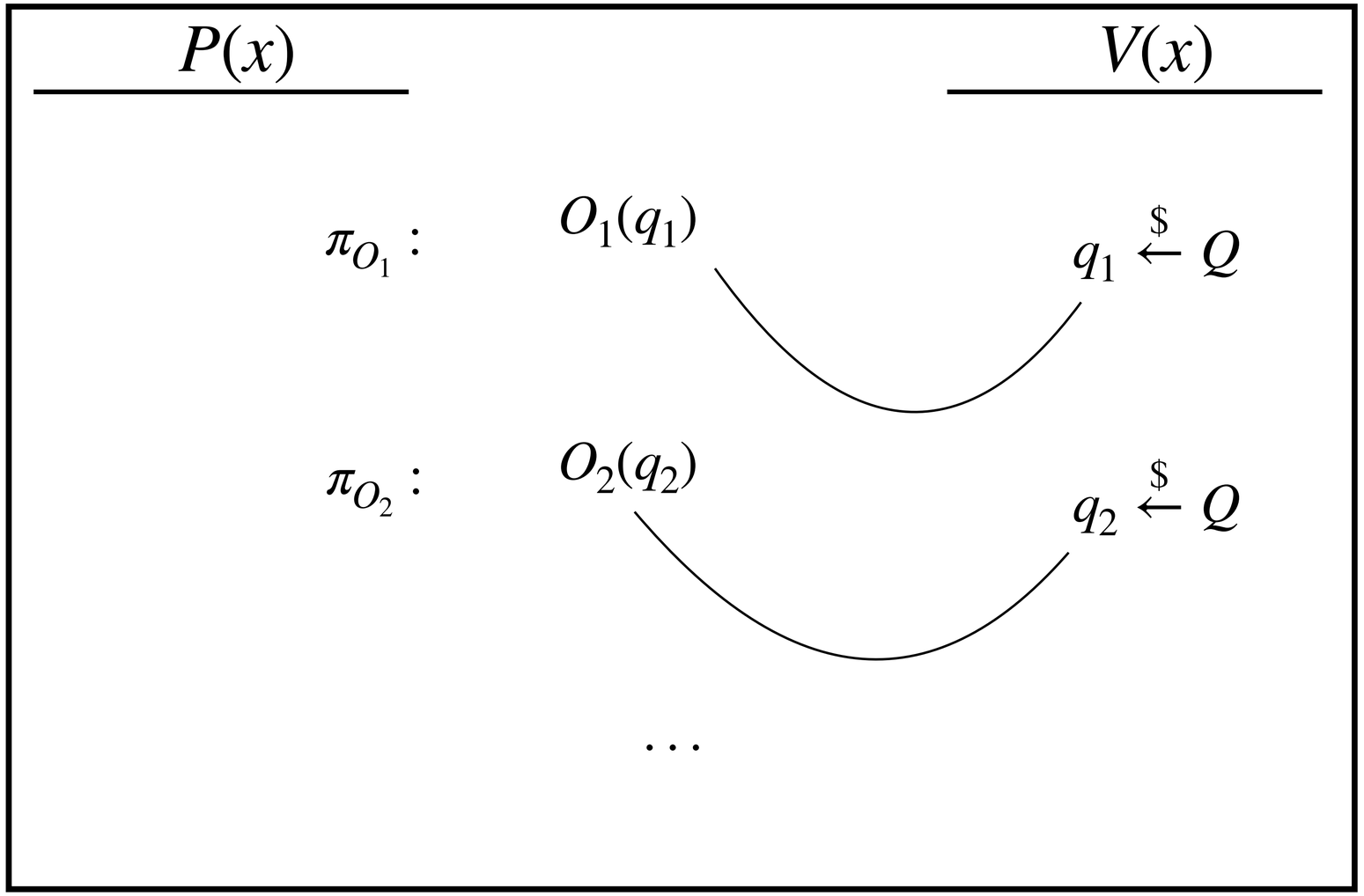
PCP

Polynomial Commitment



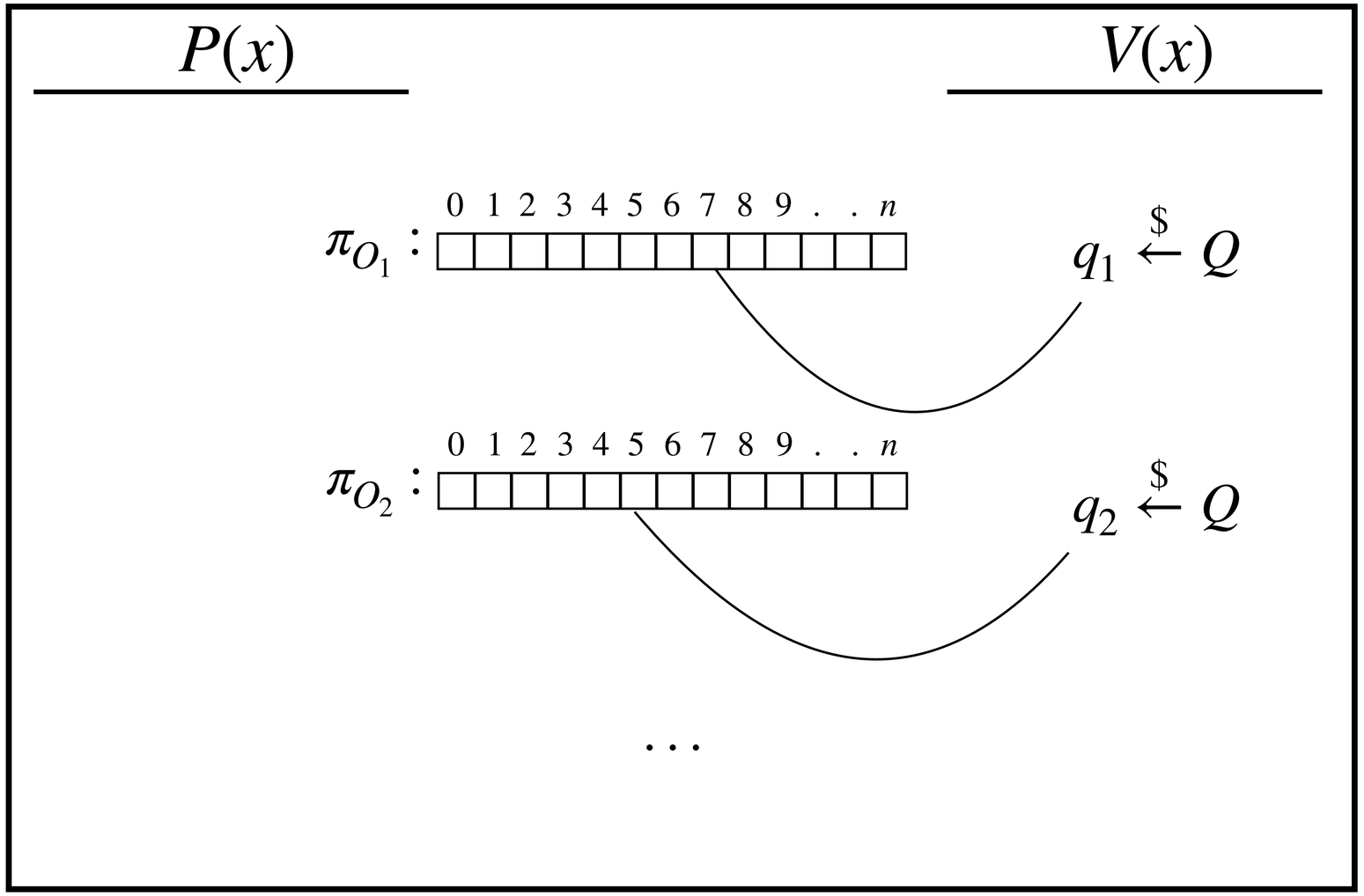
IOP

[BCS'16]



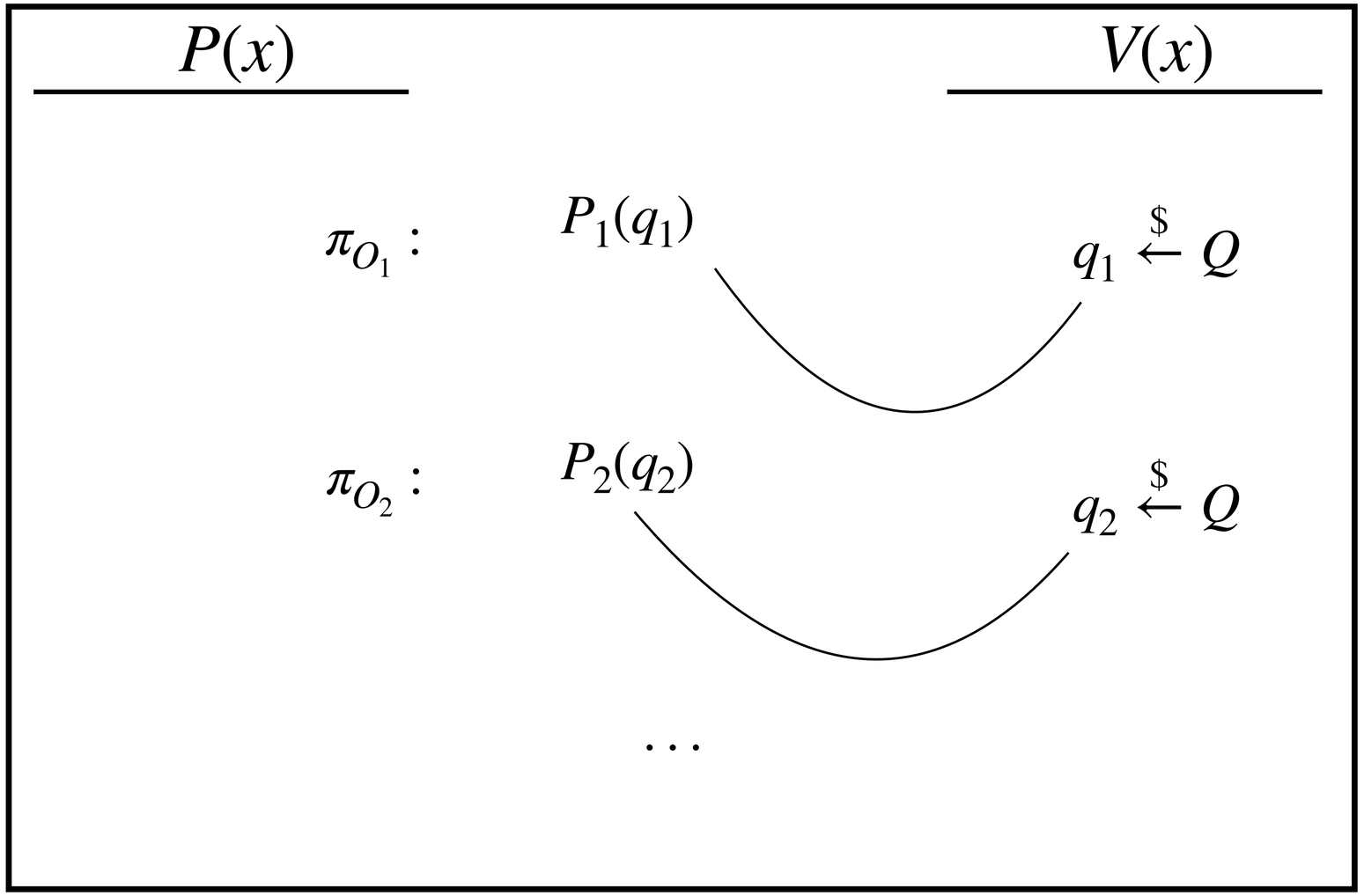
IOP

[BCS'16]


$P(x)$
$V(x)$
π_{O_1} : 0 1 2 3 4 5 6 7 8 9 . . n
$q_1 \xleftarrow{\$} Q$
π_{O_2} : 0 1 2 3 4 5 6 7 8 9 . . n
$q_2 \xleftarrow{\$} Q$
. . .

IOP

[BCS'16]



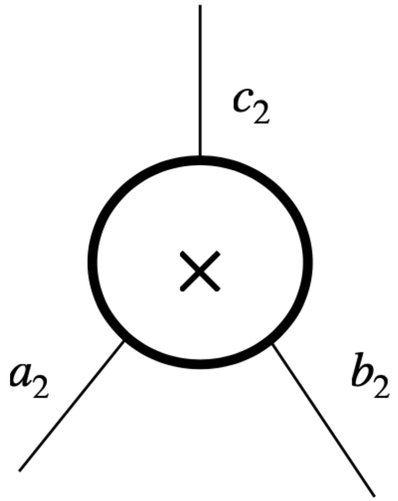
IOP Realization

- IOP + Commitment
- Most cryptographic properties inherited by the commitment scheme.
  - Trusted setup
  - Post-quantum security

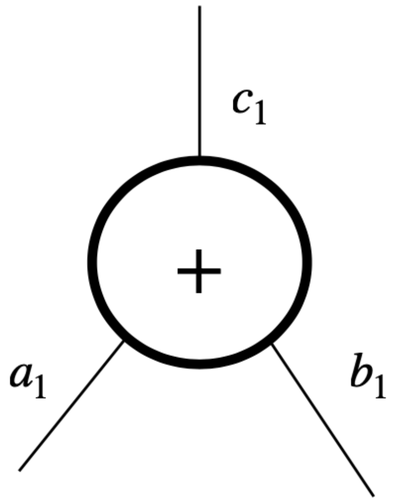
Arithmetization

Arithmetization

PLONKish



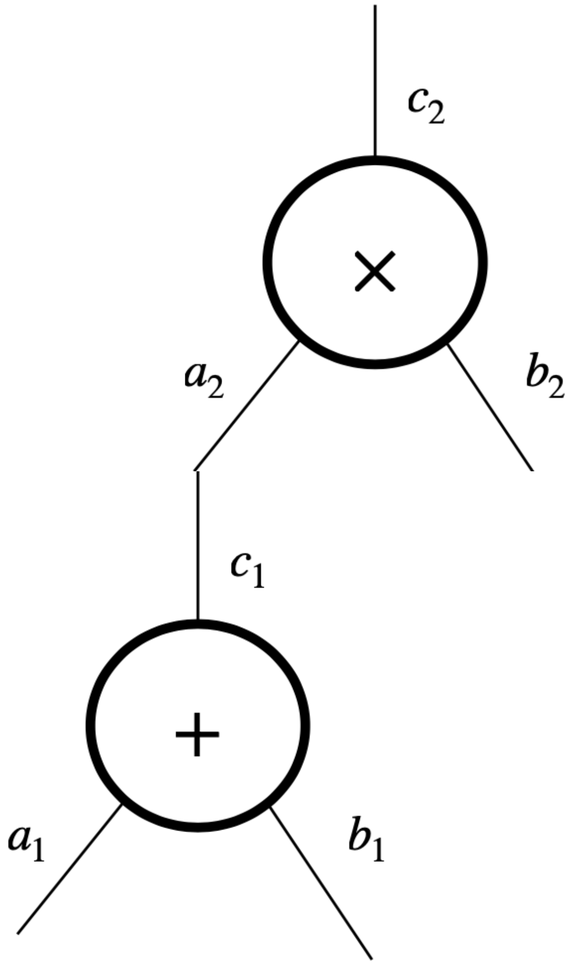
$$a_2 \cdot b_2 - c_2 = 0$$



$$a_1 + b_1 - c_1 = 0$$

Arithmetization

PLONKish



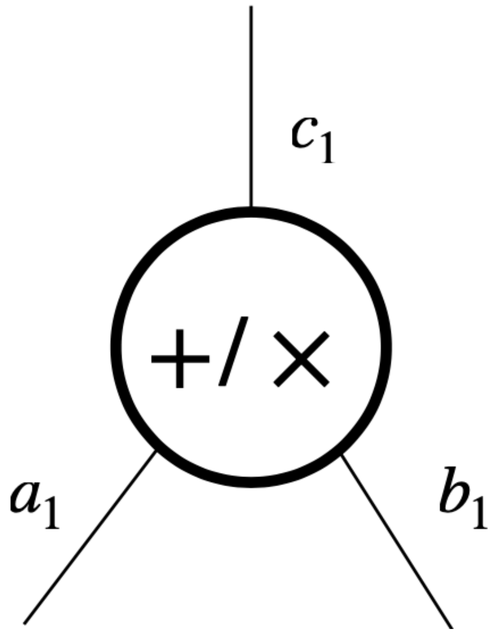
$$a_2 \cdot b_2 - c_2 = 0$$

$$a_1 + b_1 - c_1 = 0$$

$$c_1 = a_2 \text{ (Copy)}$$

Arithmetization

PLONKish



$$S \in \{0,1\}$$

$$S(a_1 + b_1) + (1 - S)(a_1 \cdot b_1) - c_1 = 0$$

Arithmetization

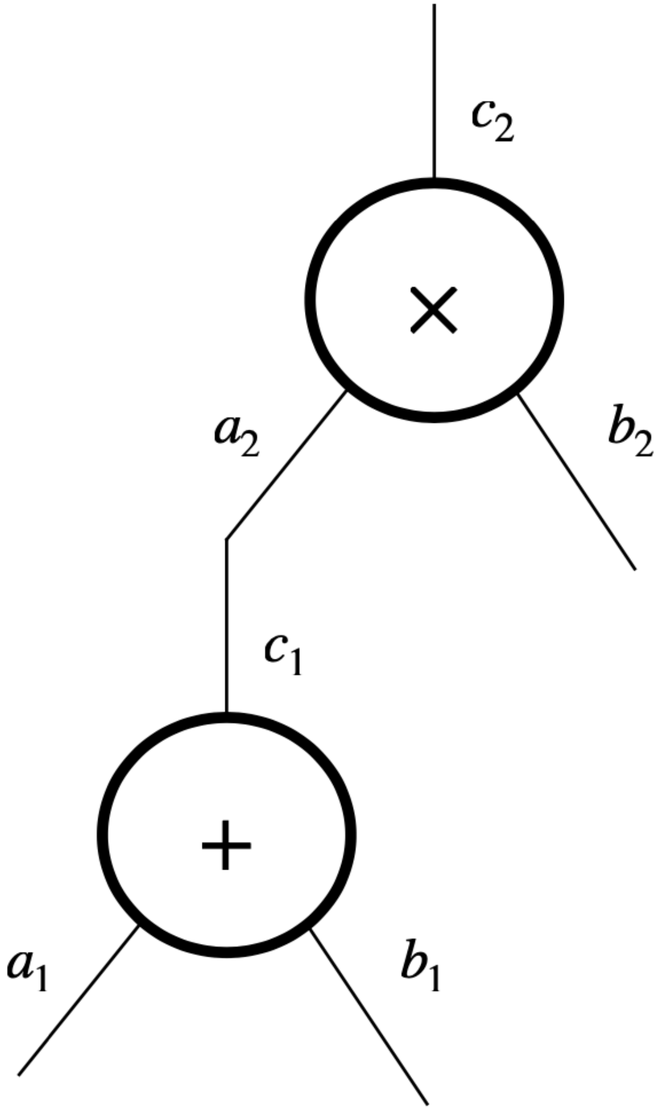
PLONKish

Computation: $(a_1 + b_1) \cdot b_2 = c_2 \mod 11$

Gate Constraints : $S_i(a_i + b_i) + (1 - S_i)(a_i \cdot b_i) - c_i = 0$

i	a_i	b_i	c_i	S_i
1	a_1	b_1	c_1	S_1
2	a_2	b_2	c_2	S_2

+ Copy



Arithmetization

PLONKish

Computation: $(a_1 + b_1) \cdot b_2 = c_2 \mod 11$

Solution: $a_1 = 4, b_1 = 5, b_2 = 10, c_1 = 9, a_2 = 9, c_2 = 2$

Gate Constraints : $S_i(a_i + b_i) + (1 - S_i)(a_i \cdot b_i) - c_i = 0$

i	a_i	b_i	c_i	S_i
1	4	5	9	1
2	9	10	2	0

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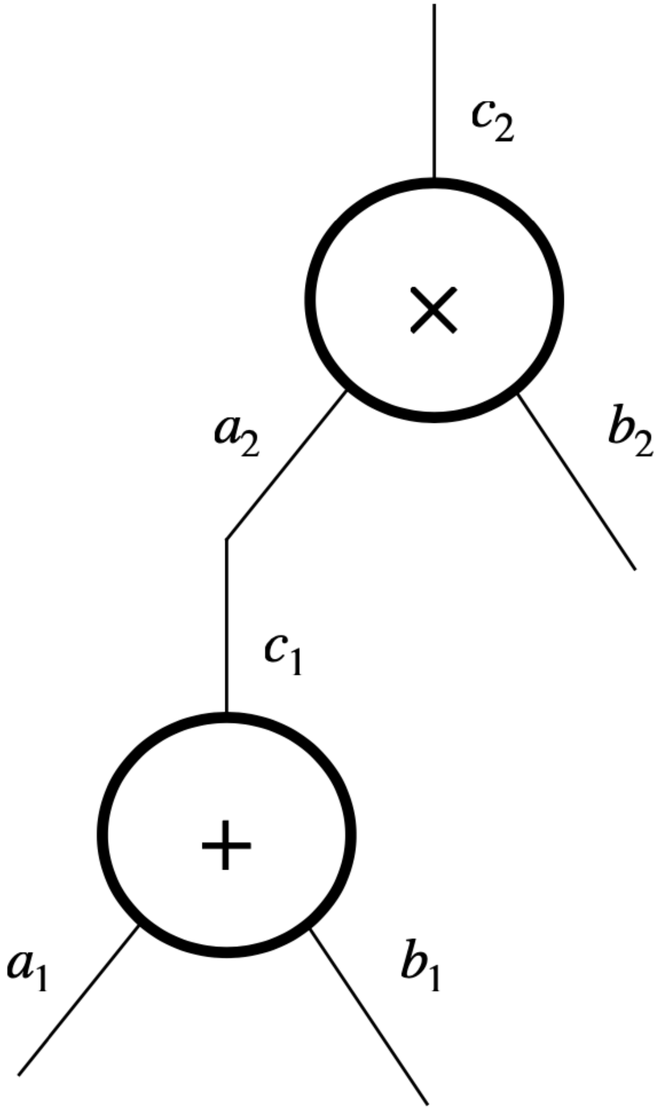


Arithmetization

PLONKish

Computation: $(a_1 + b_1) \cdot b_2 = c_2 \mod 11$

Solution: $a_1 = 4, b_1 = 5, b_2 = 10, c_1 = 9, a_2 = 9, c_2 = 2$

Gate Constraints : $S_i(a_i + b_i) + (1 - S_i)(a_i \cdot b_i) - c_i = 0$

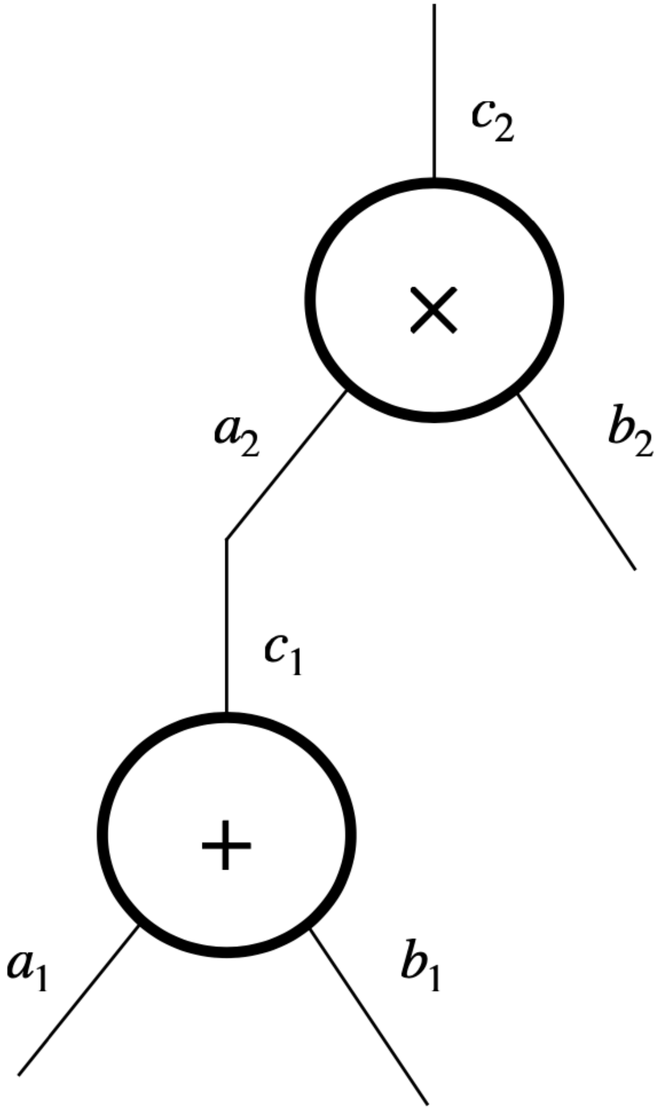
i	a_i	b_i	c_i	S_i
1	4	5	9	1
2	9	10	2	0

+ Copy

$A(x)$

$A(1) = 4$

$A(2) = 9$



Arithmetization

PLONKish

Computation: $(a_1 + b_1) \cdot b_2 = c_2 \mod 11$

Solution: $a_1 = 4, b_1 = 5, b_2 = 10, c_1 = 9, a_2 = 9, c_2 = 2$

Gate Constraints : $S_i(a_i + b_i) + (1 - S_i)(a_i \cdot b_i) - c_i = 0$

i	a_i	b_i	c_i	S_i
1	4	5	9	1
2	9	10	2	0

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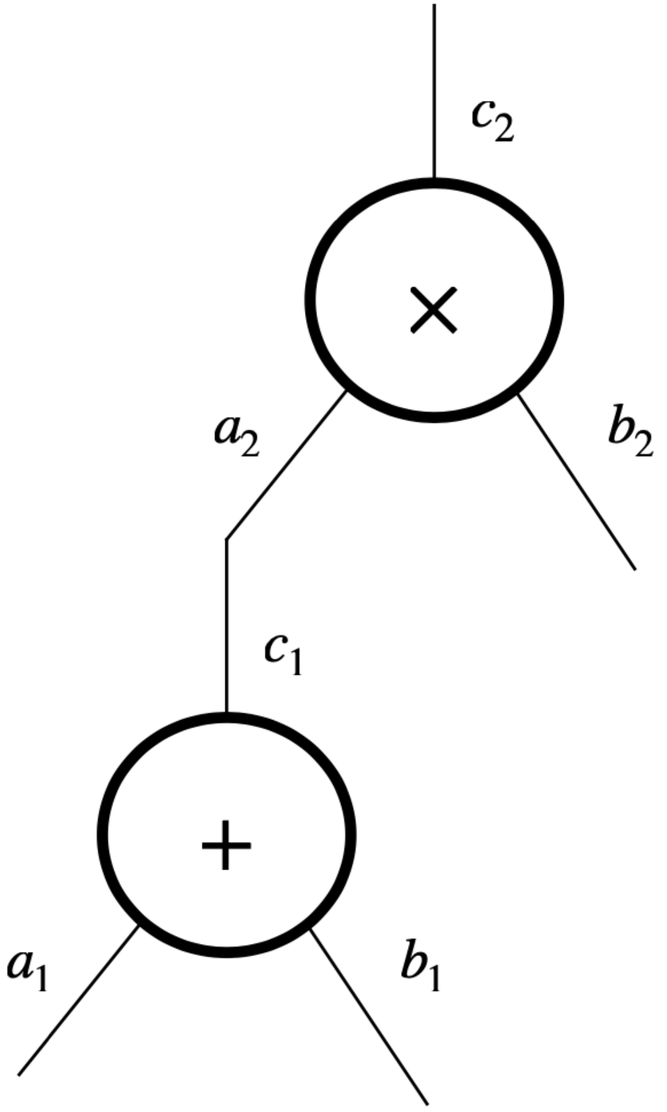

$A(x)$ $B(x)$

$A(1) = 4$ $B(1) = 5$

$A(2) = 9$ $B(2) = 10$

Arithmetization

PLONKish

Computation: $(a_1 + b_1) \cdot b_2 = c_2 \mod 11$

Solution: $a_1 = 4, b_1 = 5, b_2 = 10, c_1 = 9, a_2 = 9, c_2 = 2$

Gate Constraints : $S_i(a_i + b_i) + (1 - S_i)(a_i \cdot b_i) - c_i = 0$

i	a_i	b_i	c_i	S_i
1	4	5	9	1
2	9	10	2	0

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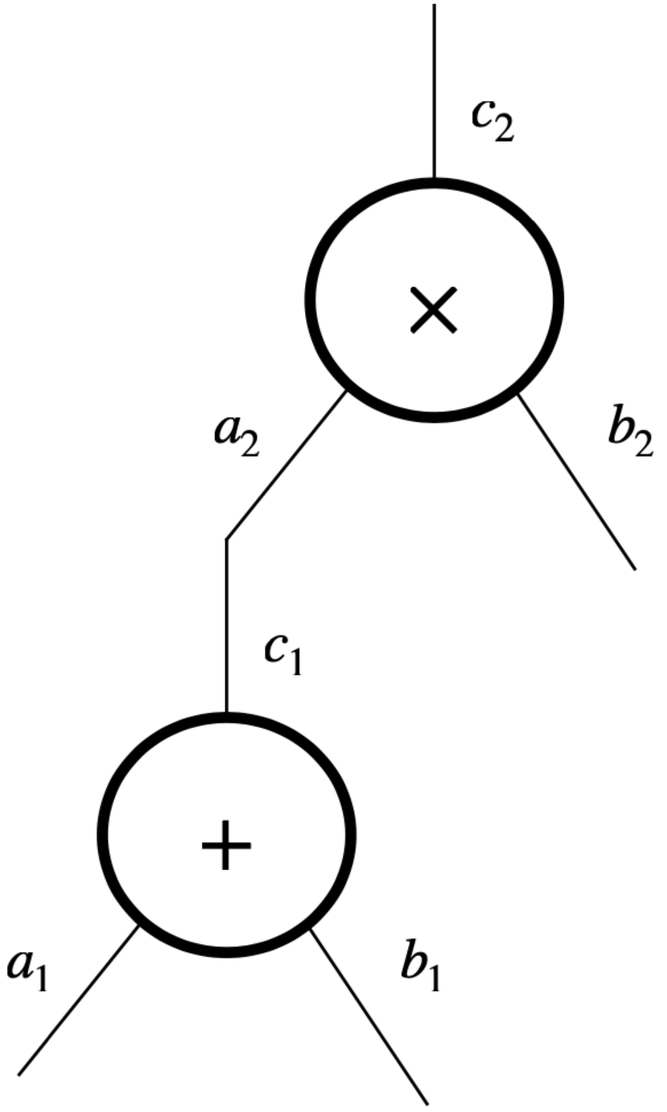

$A(x)$ $B(x)$

$A(1) = 4$ $B(1) = 5$

$A(2) = 9$ $B(2) = 10$

$$P(x) = S(x)(A(x) + B(x)) + (1 - S(x))(A(x)B(x)) - C(x)$$

Arithmetization

PLONKish

Computation: $(a_1 + b_1) \cdot b_2 = c_2 \mod 11$

Solution: $a_1 = 4,\ b_1 = 5, b_2 = 10,\ c_1 = 9,\ a_2 = 9,\ c_2 = 2$

Gate Constraints : $S_i(a_i + b_i) + (1 - S_i)(a_i \cdot b_i) - c_i = 0$

i	a_i	b_i	c_i	S_i
1	4	5	9	1
2	9	10	2	0

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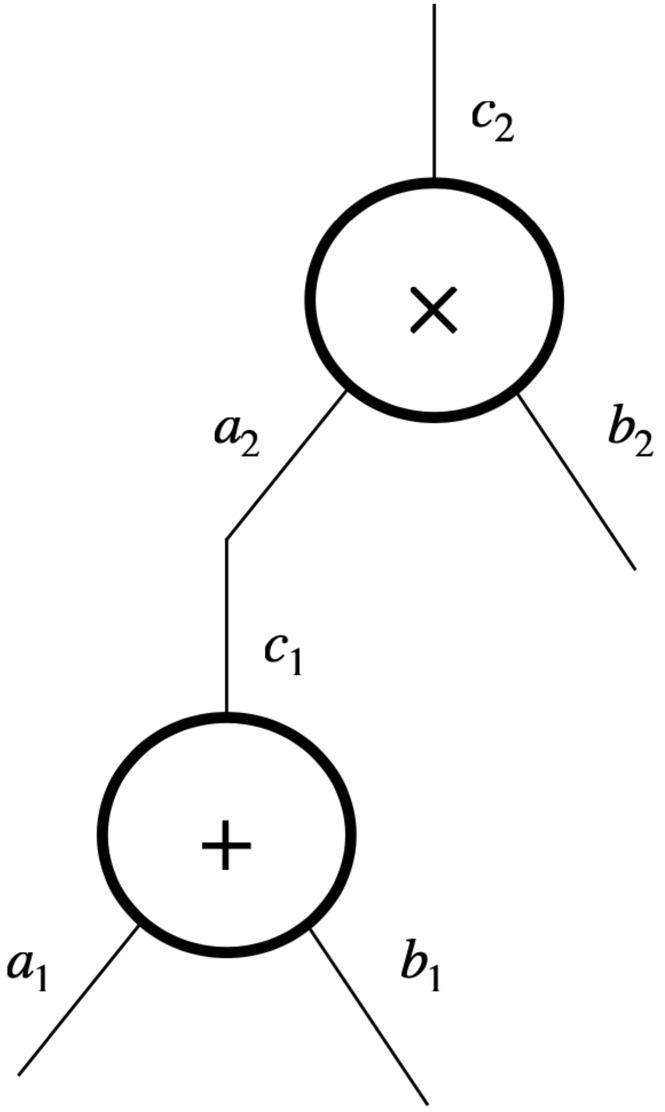


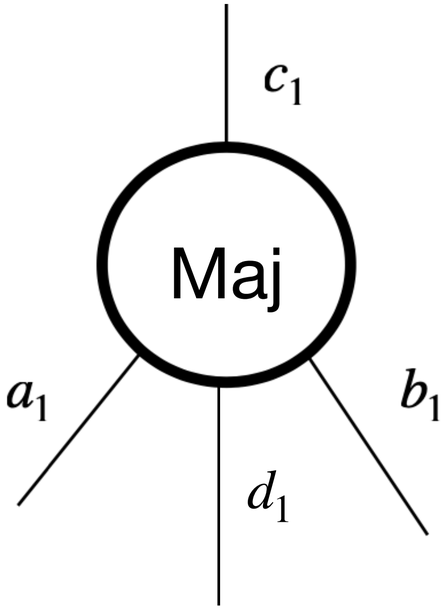
$$P(x) = S(x)(A(x) + B(x)) + (1 - S(x))(A(x)B(x)) - C(x)$$

$$P(1) = 0, \quad P(2) = 0 \implies (x - 1) \cdot (x - 2) \text{ divides } P(x)$$

Arithmetization

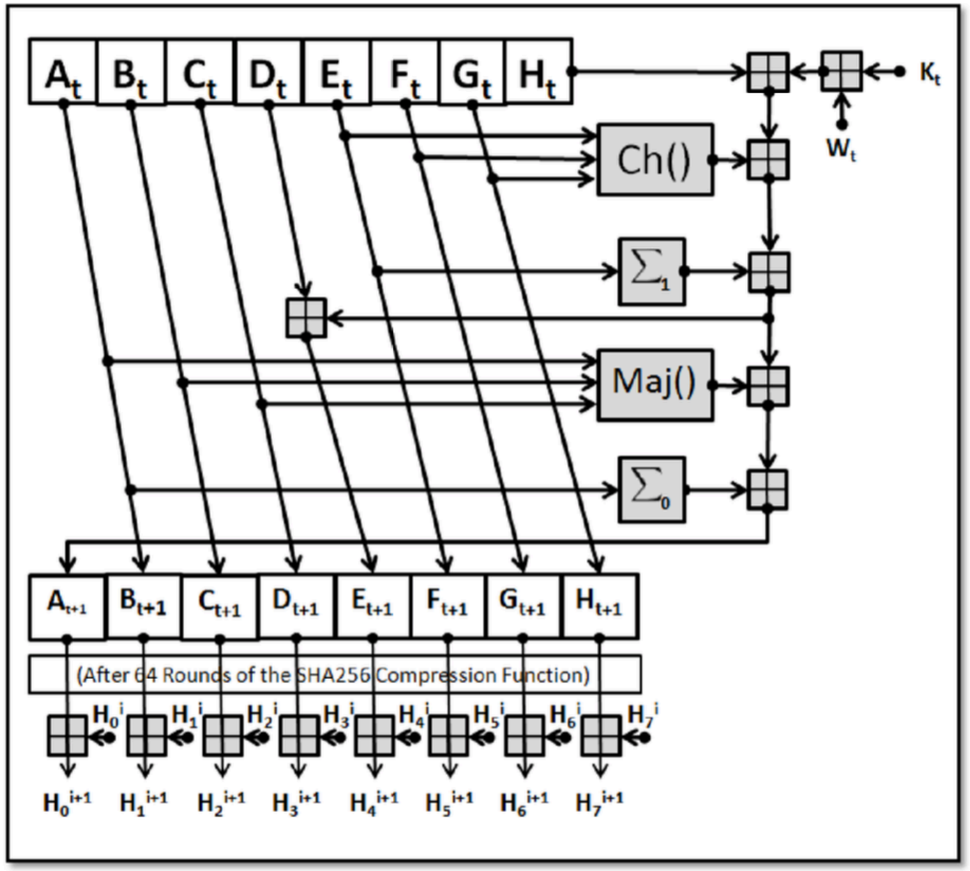
PLONKish - Custom Gates

$$S_1 \cdot (a_1 + b_1) + S_2 \cdot (a_1 \cdot b_1) + S_3 \cdot (Maj(a_1, d_1, b_1)) - c_1 = 0$$



Arithmetization

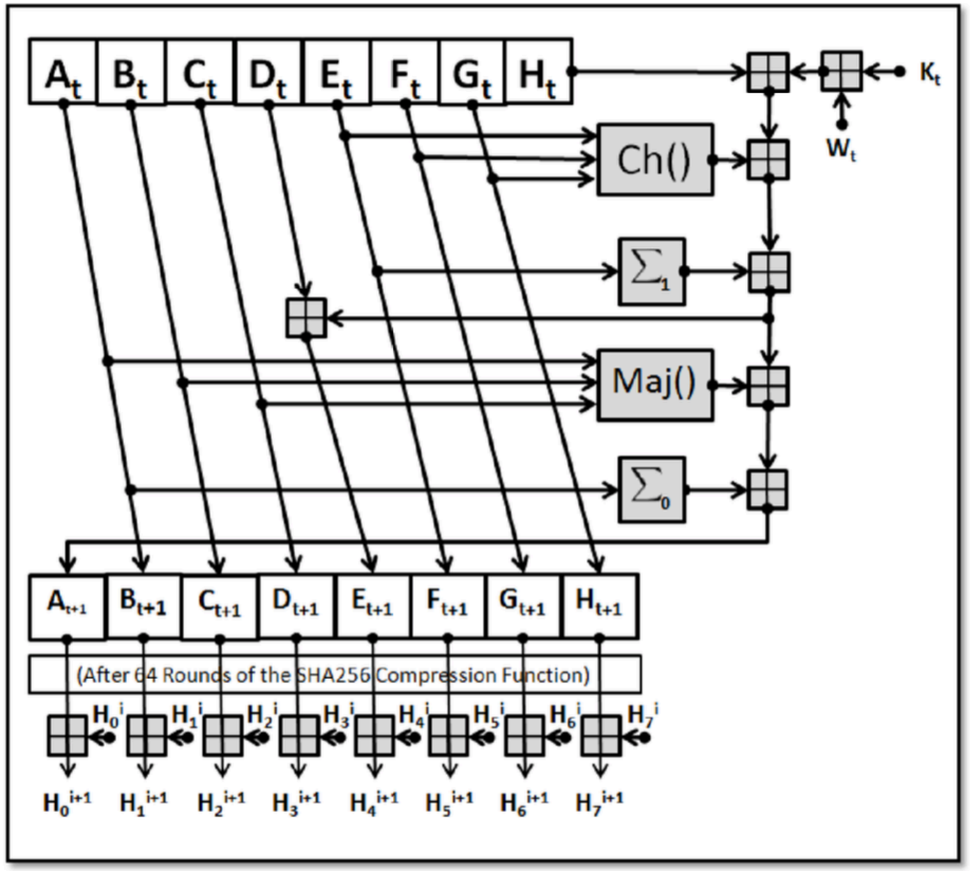
PLONKish



Alternative: Algebraic Hash Functions

Arithmetization

PLONKish - Lookup Arguments



Arithmetization

PLONKish - Lookup Arguments

a	b	c	Maj(a,b,c)
1	0	1	1

Arithmetization

PLONKish - Lookup Arguments

a	b	c	Maj(a,b,c)
1	0	1	1

a	b	c	Maj(a,b,c)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Arithmetization

PLONKish - Lookup Arguments

a	b	c	Maj(a,b,c)
1	0	1	1

a	b	c	Maj(a,b,c)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Arithmetization

PLONKish - Lookup Arguments

a	b	c	Maj(a,b,c)

a	b	c	Maj(a,b,c)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Arithmetization

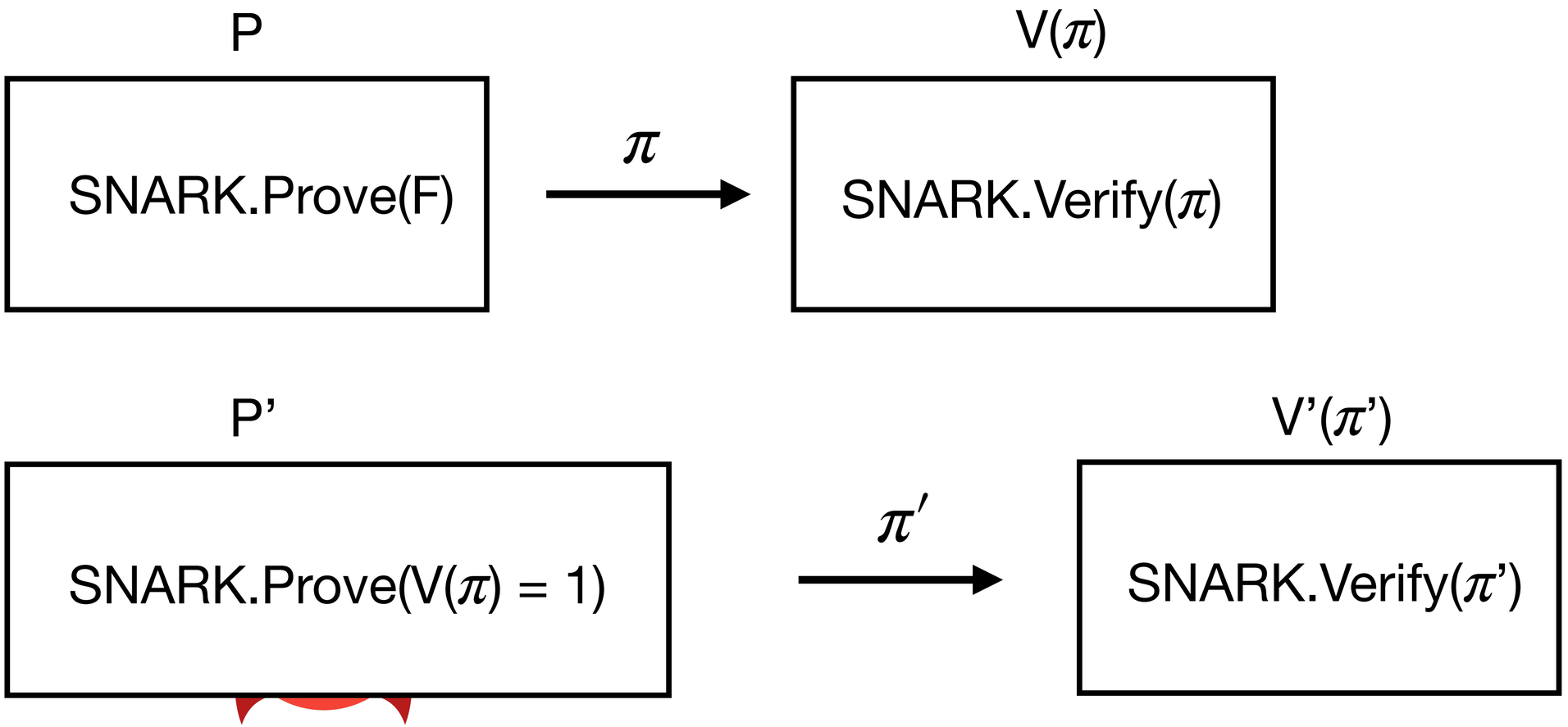
AIR - FRI

Step	R1	R2	R3
1	4	3	2
2	2	2	6
3	3	6	4
4	65	4	2

DSL

- HDL: Circom
- Zokrates, Noir, Cairo, Leo

Proof Composition

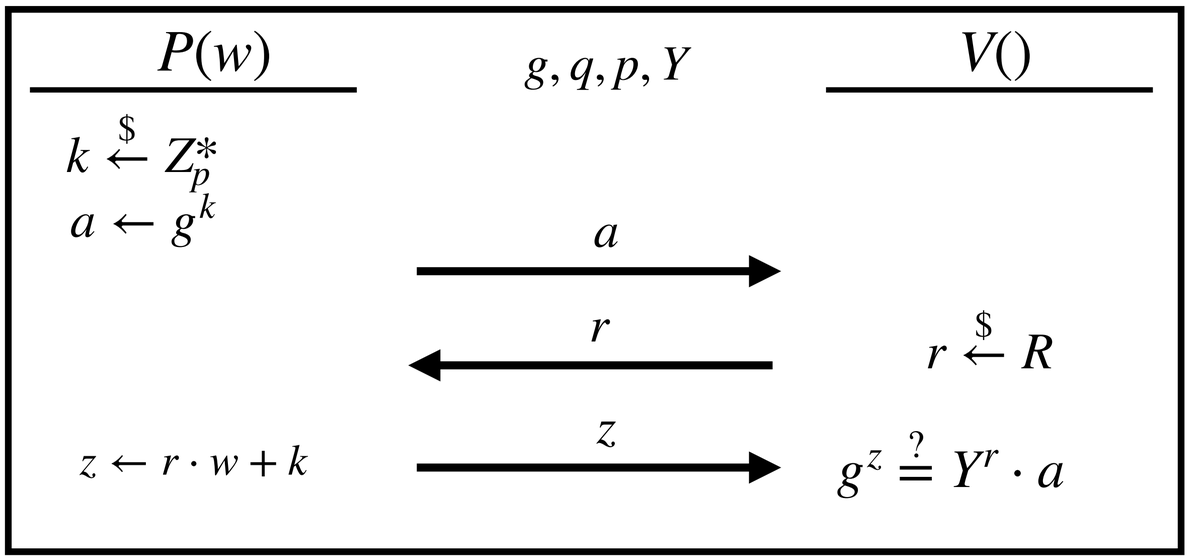


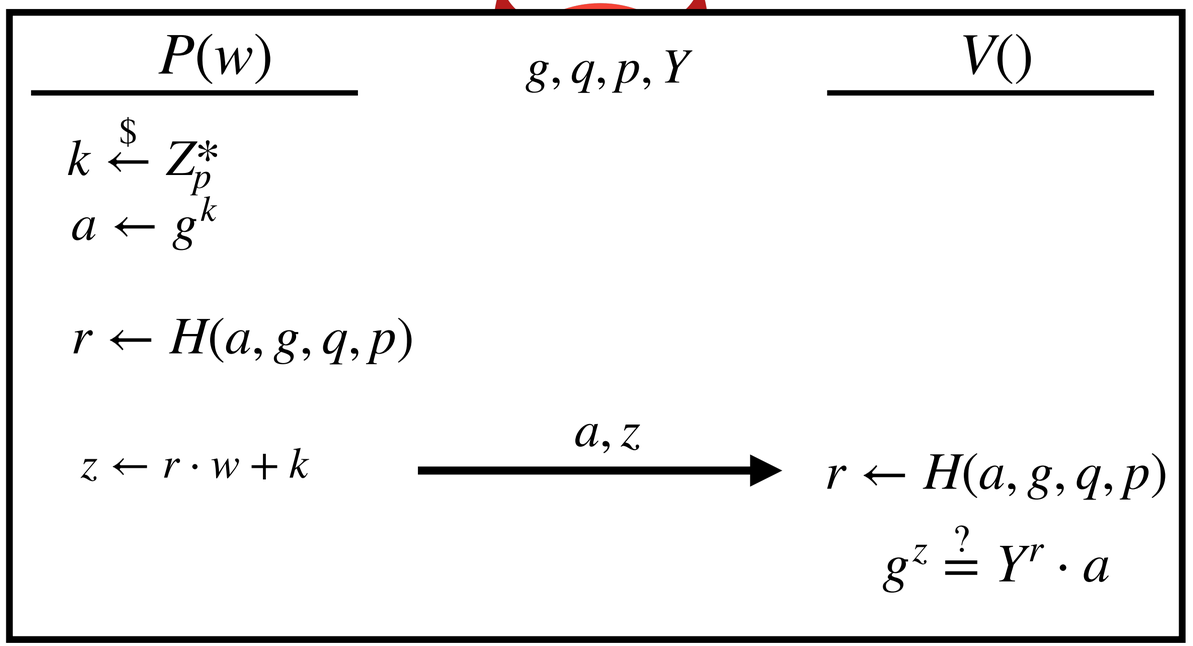
Doğru giden birçok şey var.

Ne ters gidebilir?

Sigma Protocol

Non-interactivity via Fiat Shamir





Sigma Protocol

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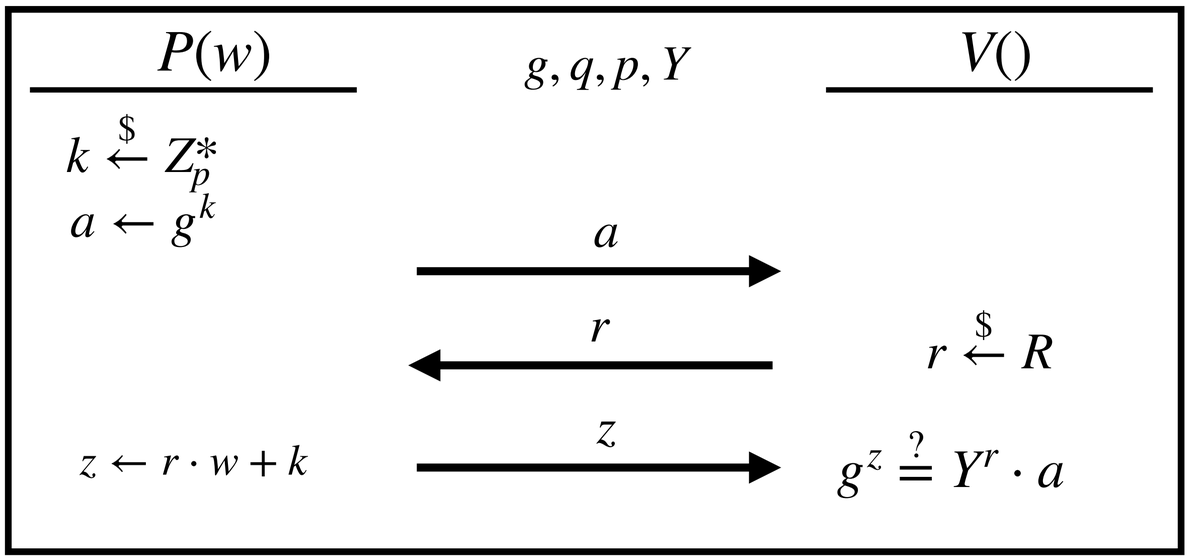

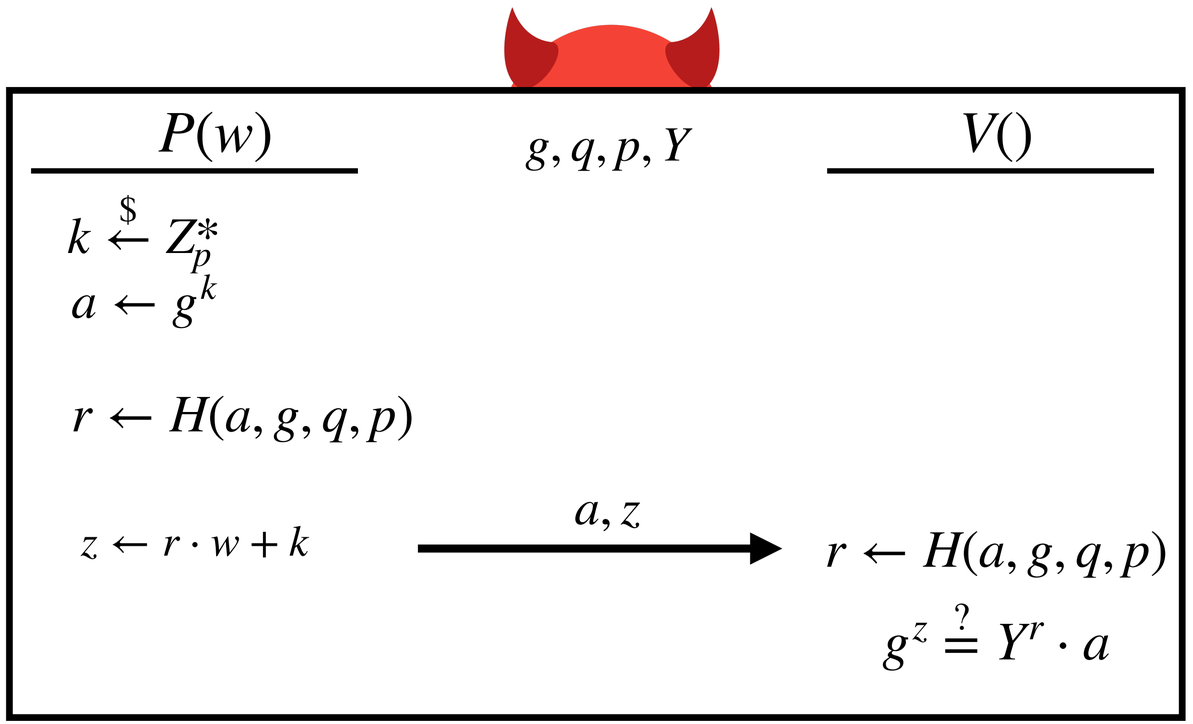

Sigma Protocol

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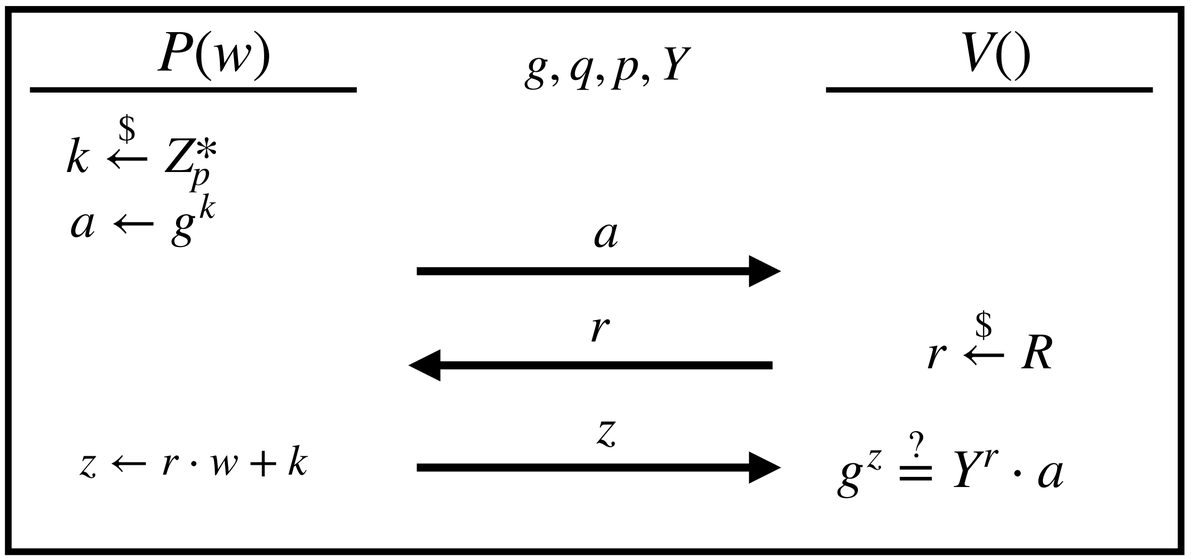

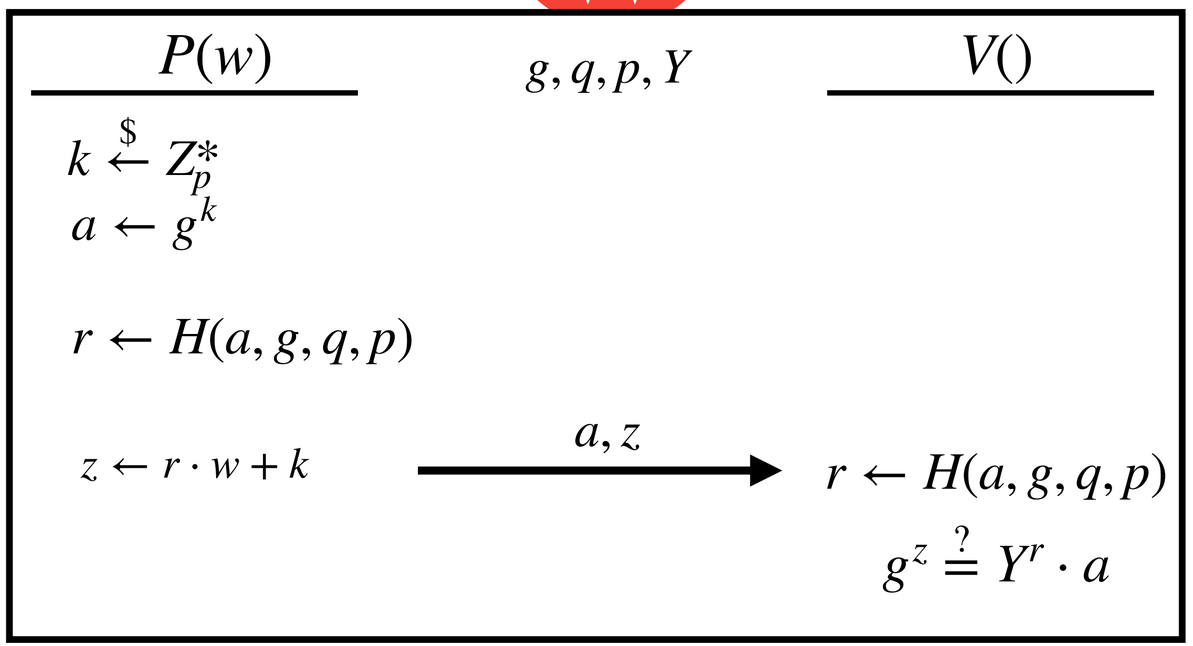

Sigma Protocol

Non-interactivity via Fiat Shamir [BPW'16]

$P^*()$	g, q, p, Y'	$V()$
$a \leftarrow K$ random public key		
$z \xleftarrow{\$} Z_p^*$		
$r \leftarrow H(a, g, q, p)$		
$Y' \leftarrow (g^z / K)^{1/r}$	$a, z \longrightarrow$	$r \leftarrow H(a, g, q, p)$
		$g^z \stackrel{?}{=} Y'^r \cdot a$

$P(w)$	g, q, p, Y	$V()$
$k \xleftarrow{\$} Z_p^*$		
$a \leftarrow g^k$		
$r \leftarrow H(a, g, q, p)$		
$z \leftarrow r \cdot w + k$	$a, z \longrightarrow$	$r \leftarrow H(a, g, q, p)$
		$g^z \stackrel{?}{=} Y^r \cdot a$

IOP Realization

- IOP + Commitment
- Most cryptographic properties inherited by the commitment scheme.
  - Trusted setup
  - Post-quantum security

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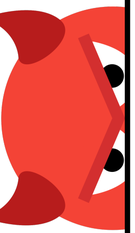
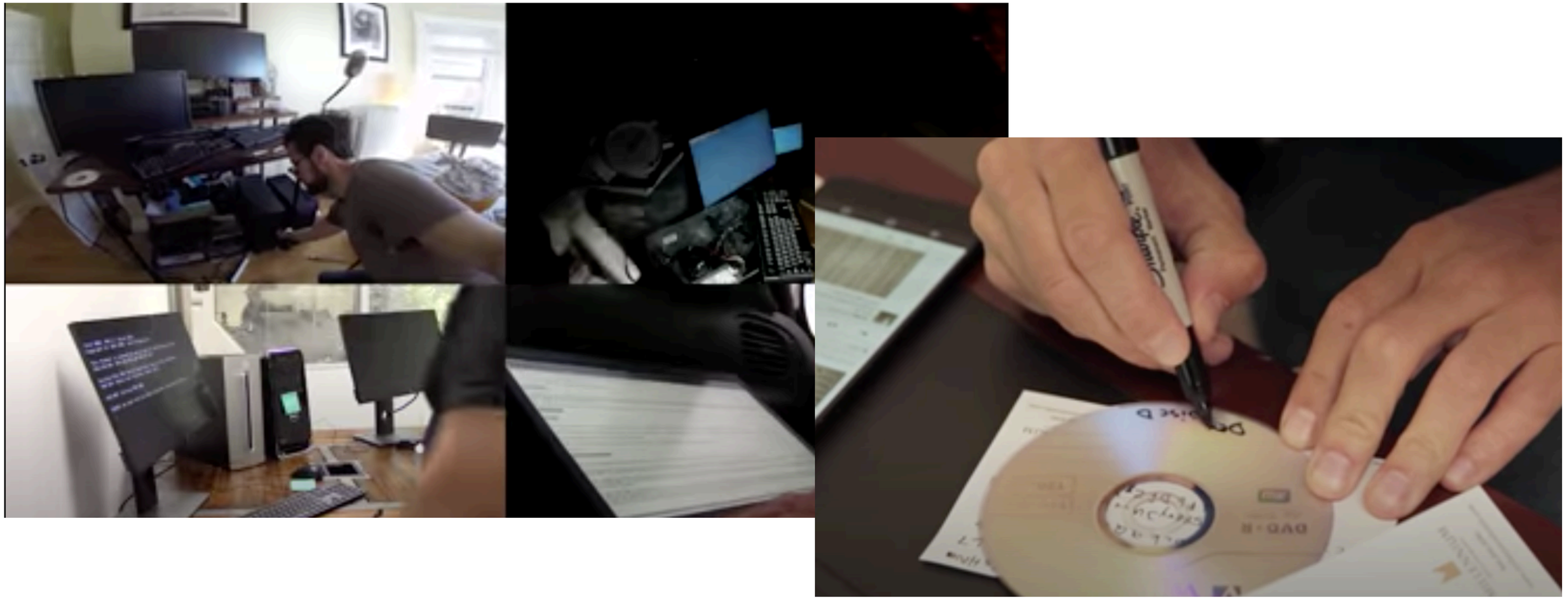


Infinite Inflation Bug

Zcash Trusted Setup (2017)



Infinite Inflation Bug

Zcash Trusted Setup (2017)

Zcash Counterfeiting Vulnerability Successfully Remediated

Josh Swihart, Benjamin Winston and Sean Bowe | February 5, 2019

Infinite Inflation Bug

Zcash Trusted Setup

BCTV'13

3. Set $\mathsf{pk} := (C, \mathsf{pk}_{\mathsf{A}}, \mathsf{pk}'_{\mathsf{A}}, \mathsf{pk}_{\mathsf{B}}, \mathsf{pk}'_{\mathsf{B}}, \mathsf{pk}_{\mathsf{C}}, \mathsf{pk}'_{\mathsf{C}}, \mathsf{pk}_{\mathsf{K}}, \mathsf{pk}_{\mathsf{H}})$ where for $i = 0,1,\ldots,m+3$:

$$\mathsf{pk}_{\mathsf{A},i} := A_i(\tau)\rho_{\mathsf{A}}\mathcal{P}_1\,, \quad \mathsf{pk}'_{\mathsf{A},i} := A_i(\tau)\alpha_{\mathsf{A}}\rho_{\mathsf{A}}\mathcal{P}_1\,,$$

$$\mathsf{pk}_{\mathsf{B},i} := B_i(\tau)\rho_{\mathsf{B}}\mathcal{P}_2\,, \quad \mathsf{pk}'_{\mathsf{B},i} := B_i(\tau)\alpha_{\mathsf{B}}\rho_{\mathsf{B}}\mathcal{P}_1\,,$$

$$\mathsf{pk}_{\mathsf{C},i} := C_i(\tau)\rho_{\mathsf{A}}\rho_{\mathsf{B}}\mathcal{P}_1\,, \mathsf{pk}'_{\mathsf{C},i} := C_i(\tau)\alpha_{\mathsf{C}}\rho_{\mathsf{A}}\rho_{\mathsf{B}}\mathcal{P}_1\,,$$

$$\mathsf{pk}_{\mathsf{K},i} := \beta\big(A_i(\tau)\rho_{\mathsf{A}} + B_i(\tau)\rho_{\mathsf{B}} + C_i(\tau)\rho_{\mathsf{A}}\rho_{\mathsf{B}}\big)\mathcal{P}_1\,,$$

Infinite Inflation Bug

Zcash Trusted Setup

BCTV'13

3. Set $\mathsf{pk} := (C, \mathsf{pk}_\mathsf{A}, \mathsf{pk}'_\mathsf{A}, \mathsf{pk}_\mathsf{B}, \mathsf{pk}'_\mathsf{B}, \mathsf{pk}_\mathsf{C}, \mathsf{pk}'_\mathsf{C}, \mathsf{pk}_\mathsf{K}, \mathsf{pk}_\mathsf{H})$ where for $i = 0,1,\ldots,m+3$:

$$\mathsf{pk}_{\mathsf{A},i} := A_i(\tau)\rho_\mathsf{A}\mathcal{P}_1\,, \quad \mathsf{pk}'_{\mathsf{A},i} := A_i(\tau)\alpha_\mathsf{A}\rho_\mathsf{A}\mathcal{P}_1\,,$$
$$\mathsf{pk}_{\mathsf{B},i} := B_i(\tau)\rho_\mathsf{B}\mathcal{P}_2\,, \quad \mathsf{pk}'_{\mathsf{B},i} := B_i(\tau)\alpha_\mathsf{B}\rho_\mathsf{B}\mathcal{P}_1\,,$$
$$\mathsf{pk}_{\mathsf{C},i} := C_i(\tau)\rho_\mathsf{A}\rho_\mathsf{B}\mathcal{P}_1\,, \mathsf{pk}'_{\mathsf{C},i} := C_i(\tau)\alpha_\mathsf{C}\rho_\mathsf{A}\rho_\mathsf{B}\mathcal{P}_1\,,$$
$$\mathsf{pk}_{\mathsf{K},i} := \beta\big(A_i(\tau)\rho_\mathsf{A} + B_i(\tau)\rho_\mathsf{B} + C_i(\tau)\rho_\mathsf{A}\rho_\mathsf{B}\big)\mathcal{P}_1\,,$$

BCTV'19

3. Set $\mathsf{pk} := (C, \mathsf{pk}_\mathsf{A}, \mathsf{pk}'_\mathsf{A}, \mathsf{pk}_\mathsf{B}, \mathsf{pk}'_\mathsf{B}, \mathsf{pk}_\mathsf{C}, \mathsf{pk}'_\mathsf{C}, \mathsf{pk}_\mathsf{K}, \mathsf{pk}_\mathsf{H})$ where:

$$\mathsf{pk}_\mathsf{A} := \{A_i(\tau)\rho_\mathsf{A}\mathcal{P}_1\}_{i=0}^{m+3}\,, \quad \mathsf{pk}'_\mathsf{A} := \{A_i(\tau)\alpha_\mathsf{A}\rho_\mathsf{A}\mathcal{P}_1\}_{i=n+1}^{m+3}$$
$$\mathsf{pk}_\mathsf{B} := \{B_i(\tau)\rho_\mathsf{B}\mathcal{P}_2\}_{i=0}^{m+3}\,, \quad \mathsf{pk}'_\mathsf{B} := \{B_i(\tau)\alpha_\mathsf{B}\rho_\mathsf{B}\mathcal{P}_1\}_{i=0}^{m+3}\,,$$
$$\mathsf{pk}_\mathsf{C} := \{C_i(\tau)\rho_\mathsf{A}\rho_\mathsf{B}\mathcal{P}_1\}_{i=0}^{m+3}\,, \mathsf{pk}'_\mathsf{C} := \{C_i(\tau)\alpha_\mathsf{C}\rho_\mathsf{A}\rho_\mathsf{B}\mathcal{P}_1\}_{i=0}^{m+3}\,,$$
$$\mathsf{pk}_\mathsf{K} := \{\beta\big(A_i(\tau)\rho_\mathsf{A} + B_i(\tau)\rho_\mathsf{B} + C_i(\tau)\rho_\mathsf{A}\rho_\mathsf{B}\big)\mathcal{P}_1\}_{i=0}^{m+3}\,,$$

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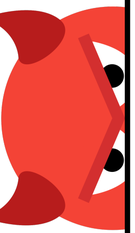
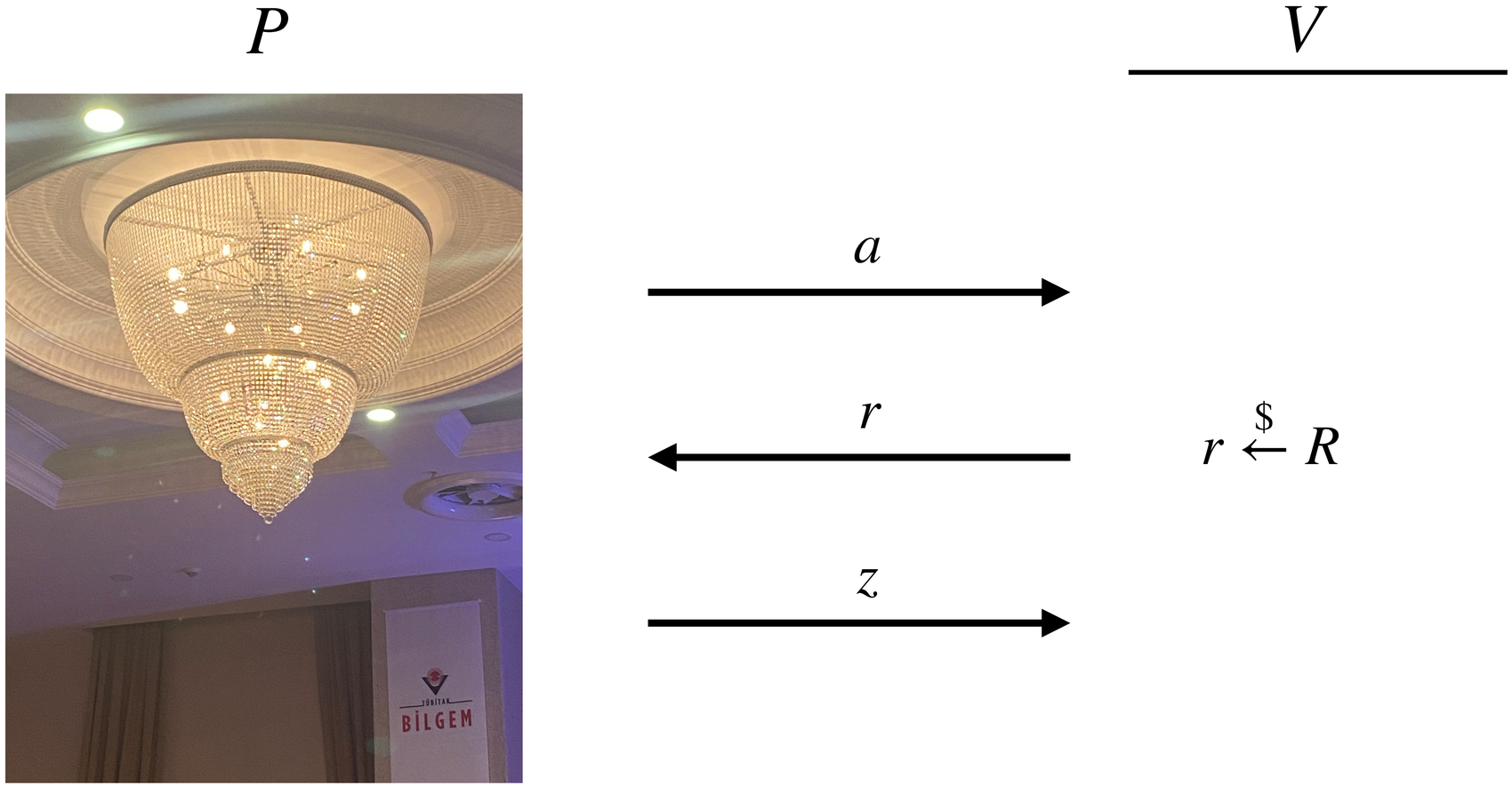


Quantum Soundness

Quantum Rewinding [LWS'22]



Quantum Soundness

Quantum Rewinding [LWS'22]

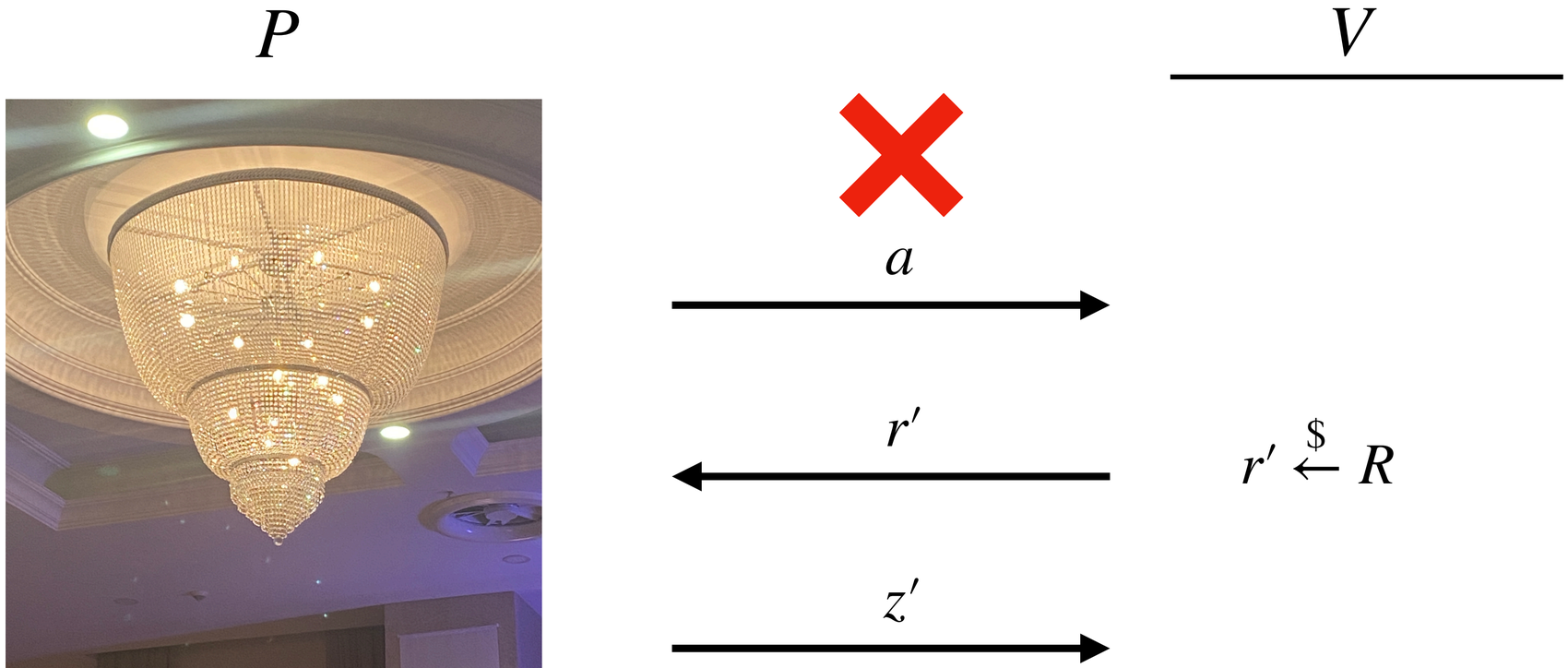
P

a

V

Quantum Soundness

Quantum Rewinding [LWS'22]



Quantum Soundness

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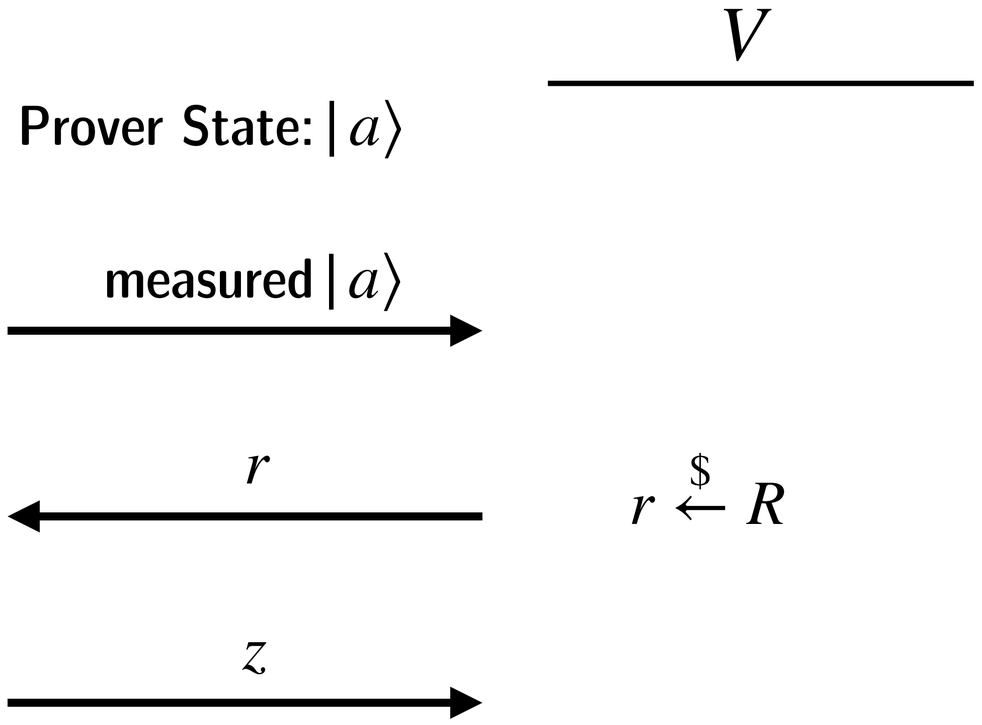

Quantum Soundness

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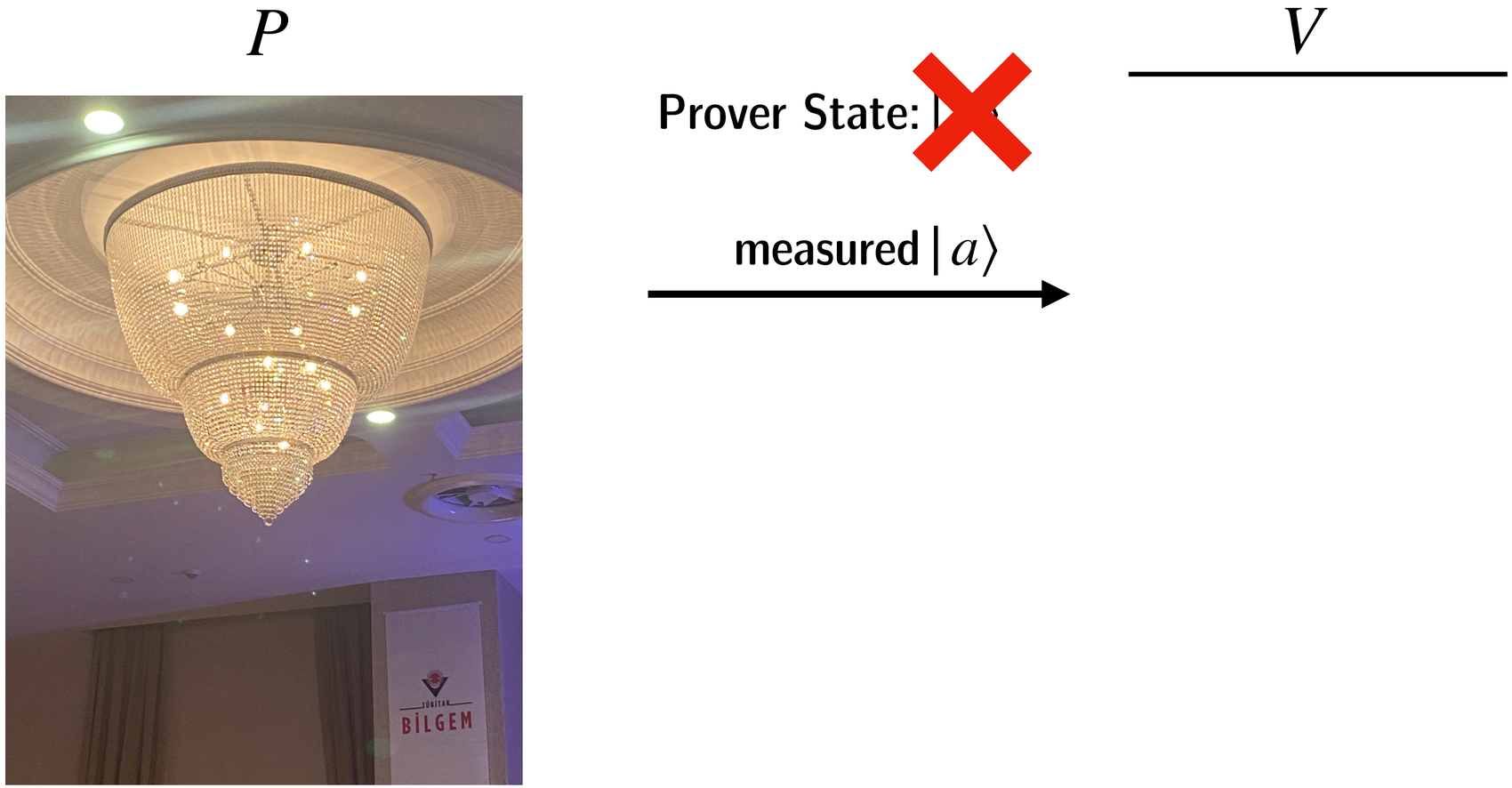
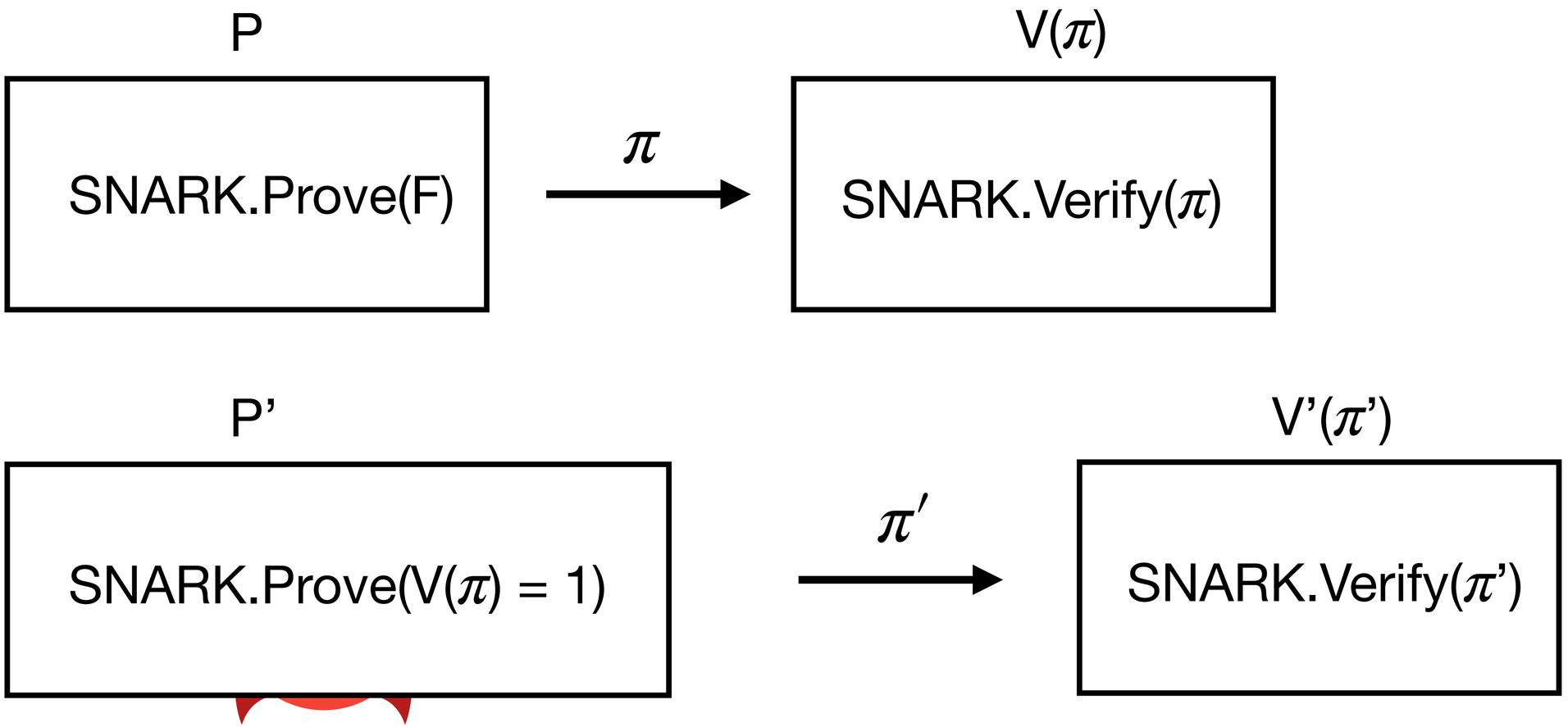
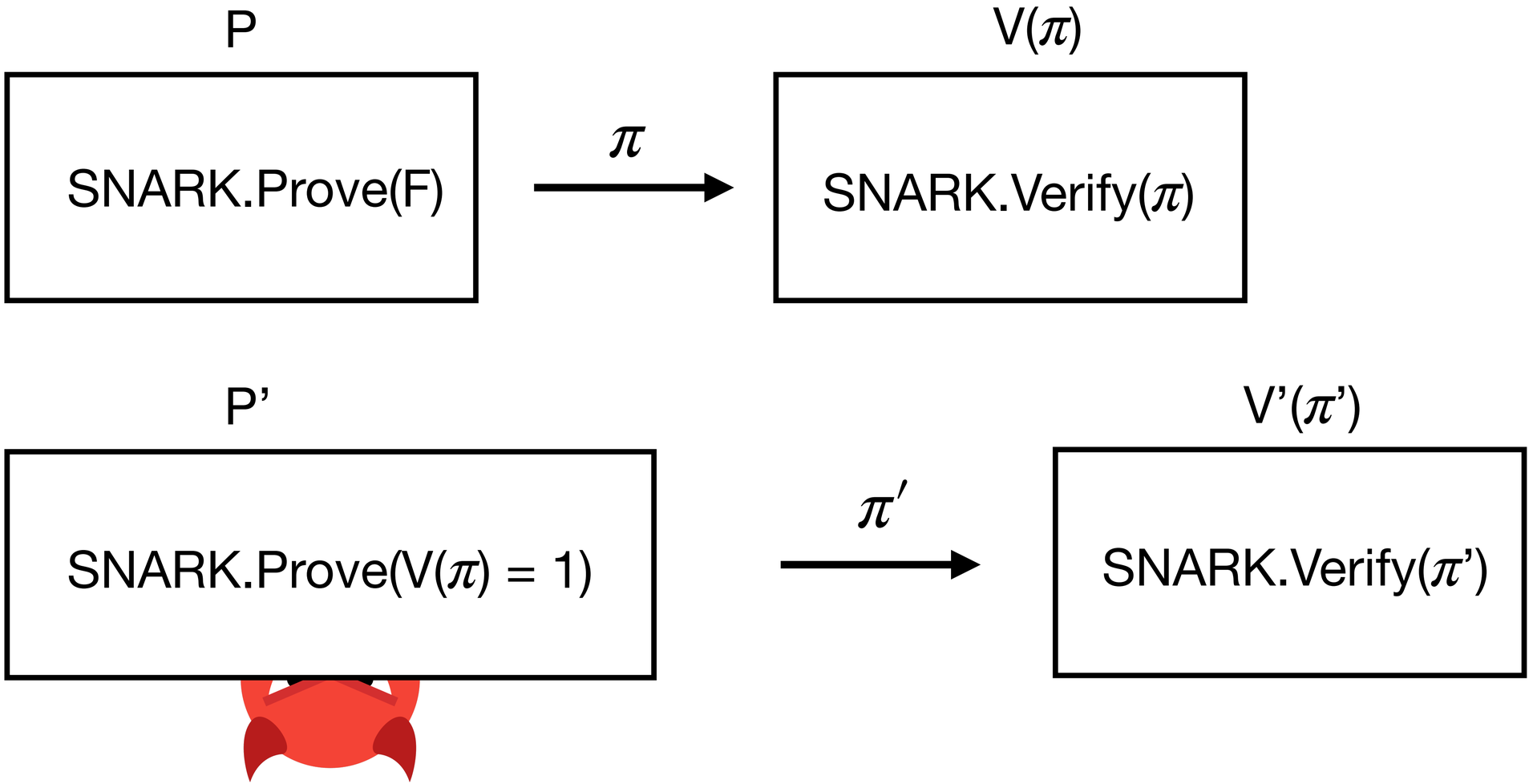


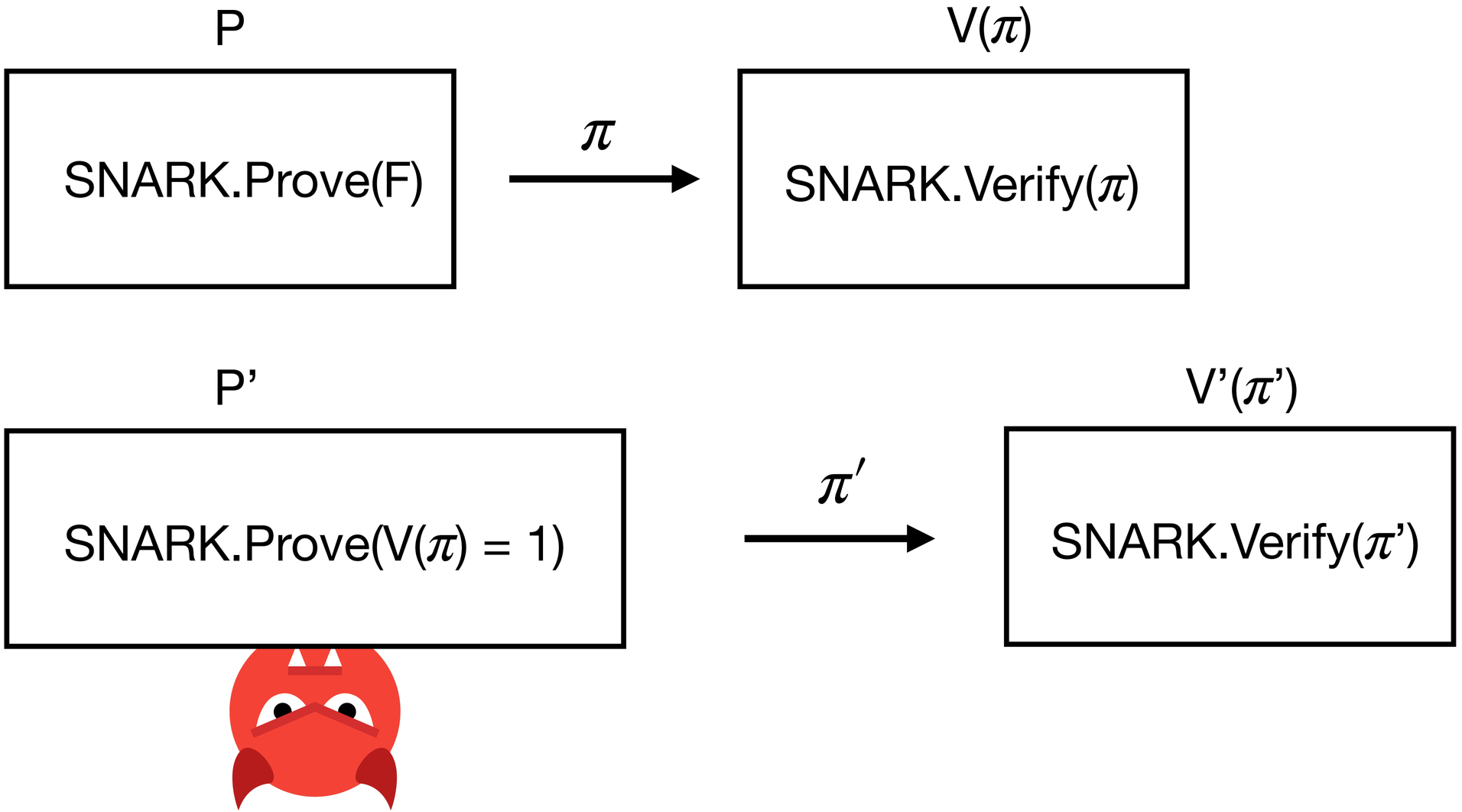
Proof Composition



Proof Composition



Proof Composition



Doğru giden birçok şey var

$\wedge$

Ters gidebilecek birçok şey var

$\Longrightarrow$

Birçok şey ters gidecek

Teşekkürler!

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abdullah.talayhan@epfl.ch